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Ground state of hydrogen as a zero-point-fluctuation-determined state

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We show here that, within the stochastic electrodynamic formulation and at the level of Bohr theory, the ground state of the hydrogen atom can be precisely defined as resulting from a dynamic equilibrium between radiation emitted due to acceleration of the electron in its ground-state orbit and radiation absorbed from zero-point fluctuations of the background vacuum electromagnetic field, thereby resolving the issue of radiative collapse of the Bohr atom.

INTRODUCTION

The key role played by zero-point fluctuations (ZPF's) in determining the behavior of certain quantum systems is such that the systems in question might properly be called ZPF determined. A simple but elegant example of a ZPF-determined system is provided by the attractive quantum force (Casimir force) between conducting parallel plates,[1–4] where the force results from the redistribution of viable normal modes (and hence in the associated vacuum electromagnetic ZPF energy) as the distance between the plates changes.

Although essentially thought of in quantum-mechanical terms, ZPF-determined systems have successfully yielded to the analysis techniques of stochastic electrodynamics[5,6] (SED), which treat quantum field-particle interactions on the basis of classical concepts. In the SED formulation one assumes that charged point-mass particles interact with a background of random classical electromagnetic zero-point radiation with energy spectrum (as in the quantum-mechanical case)

$$\rho(\omega)d\omega=\frac{\hbar\omega^3}{2\pi^2c^3}d\omega\ , \tag{1}$$

which corresponds to an average energy $\frac{1}{2}\hbar\omega$ per normal mode.

It is of special interest that, as shown by Marshall[7] and Boyer,[8] the spectrum's cubic dependence on frequency can be derived solely on the basis of Lorentz invariance, since this spectrum is unique in its property that delicate cancellation of Doppler shifts with velocity boosts leaves the spectrum Lorentz invariant. We also note that (within SED) $\hbar$ appears in the above expression simply in the role of scale factor to align theory with experimental results; no quantum interpretation is required in its application, and all other appearances of $\hbar$ in the theory can be traced back to its appearance in this expression. Thus, the starting point for the SED formulation is seen to be parsimonious.

Topics successfully analyzed within the SED formulation (i.e., yielding precise quantitative agreement with QED treatments) include the Planck blackbody radiation spectrum,[8] Casimir[3,4,9] and van der Waals forces,[10] and the thermal effects of acceleration through the vacuum[11] (to name a few), all originally thought to be soluble only within the quantum formalism. We shall apply here the same SED techniques that have been used successfully in these previous studies. We also outline how the corresponding QED treatment leads to identical results.

Of particular pedagogical interest for the insight it sheds on ZPF modeling within the SED framework is the determination of the ground state of the hydrogen atom. It is hypothesized that (at the level of Bohr theory) the ground-state orbit is a ZPF-determined state, determined by a balance between radiation emitted due to acceleration of the electron and radiation absorbed from the zero-point background. In an early heuristic derivation by Boyer,[12] a result $m\omega r^2=\frac{3}{4}\hbar$ was obtained; this was considered close enough (for the approach used) to the desired $m\omega r^2=\hbar$ to indicate the basic soundness of the hypothesis.

We present here within the framework of SED an alternative derivation that yields precisely the correct result for the ground state.

INTERACTION OF CHARGED HARMONIC OSCILLATOR WITH ZERO-POINT BACKGROUND

In SED the vacuum is assumed to be filled with random classical zero-point electromagnetic radiation which is homogeneous, isotropic, and Lorentz invariant. Written as a sum over plane waves, the random radiation can be expressed as

$$\mathbf{E}^{zp}(\mathbf{r},t)=\mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k\,\hat{\varepsilon}\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\times e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t+i\theta(\mathbf{k},\sigma)}\ , \tag{2}$$

$$\mathbf{H}^{zp}(\mathbf{r},t)=\mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k(\hat{\mathbf{k}}\times\hat{\varepsilon})\left[\frac{\hbar\omega}{8\pi^3\mu_0}\right]^{1/2}\times e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t+i\theta(\mathbf{k},\sigma)}\ , \tag{3}$$

where $\sigma=1,2$ denote orthogonal polarizations, $\hat{\varepsilon}$ and $\hat{\mathbf{k}}$ are orthogonal unit vectors in the direction of the electric field polarization and wave propagation vectors, respectively, $\theta(\mathbf{k},\sigma)$ are random phases distributed uniformly on the interval 0 to 2π (independently distributed for each

$\mathbf{k},\sigma$), and $\omega=kc$. It is an easy exercise to show that the Fourier compositions assumed in (2) and (3) correspond to the spectrum given in (1). Here, as in SED generally, the zero-point field is treated in every way as a real, physical field, and is taken to be responsible for the effects of interest.[13]

We begin our discussion of particle-field interaction by considering a one-dimensional charged harmonic oscillator of natural frequency ω_0, located at the origin and immersed in zero-point radiation. For orientation along the x axis, the (nonrelativistic) equation of motion for a particle of mass m and charge e, including radiation damping, is given by

$$m\ddot{x}+m\omega_0^2x=\left[\frac{e^2}{6\pi\epsilon_0c^3}\right]\dddot{x}+eE_x^{\rm zp}(0,t)\ . \tag{4}$$

Substitution of (2) into (4) leads to the following expressions for displacement and velocity:

$$x=\frac{e}{m}\mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k(\hat{\epsilon}\cdot\hat{\mathbf{x}})\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\frac{1}{D}\times e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t+i\theta(\mathbf{k},\sigma)}\ , \tag{5}$$

$$v=\dot{x}=\frac{e}{m}\mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k(\hat{\epsilon}\cdot\hat{\mathbf{x}})\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\left[\frac{-i\omega}{D}\right]\times e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t+i\theta(\mathbf{k},\sigma)}\ , \tag{6}$$

where

$$D=-\omega^2+\omega_0^2-i\Gamma\omega^3\ , \tag{7}$$

$$\Gamma=\frac{e^2}{6\pi\epsilon_0mc^3}\ . \tag{8}$$

From (2) and (6) we now calculate the average power absorbed from the zero-point background as

$$\begin{aligned}\langle P^{\rm abs}\rangle&=\langle\mathbf{F}\cdot\mathbf{v}\rangle\\&=\langle e\mathbf{E}^{\rm zp}\cdot\mathbf{v}\rangle\\&=\tfrac{1}{2}\mathrm{Re}\left\langle\frac{e^2}{m}\sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}\int d^3k\int d^3k'(\hat{\epsilon}\cdot\hat{\mathbf{x}})(\hat{\epsilon}'\cdot\hat{\mathbf{x}})\left[\frac{\hbar\omega}{8\pi^3\epsilon_0}\right]^{1/2}\left[\frac{\hbar\omega'}{8\pi^3\epsilon_0}\right]^{1/2}\left[\frac{i\omega'}{D^*}\right]\right.\\&\qquad\left.\times\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}-i(\omega-\omega')t+i\theta(\mathbf{k},\sigma)-i\theta(\mathbf{k}',\sigma')]\right\rangle\ ,\end{aligned} \tag{9}$$

where use of the complex conjugate and the notation $\frac{1}{2}$Re stems from the use of exponential notation. Equation (9) can, however, be simplified to

$$\langle P^{\rm abs}\rangle=\tfrac{1}{2}\mathrm{Re}\left[\frac{e^2}{m}\sum_{\sigma=1}^{2}\int d^3k(\hat{\epsilon}\cdot\hat{\mathbf{x}})^2\frac{\hbar\omega}{8\pi^3\epsilon_0}\frac{i\omega}{D^*}\right]\ , \tag{10}$$

where averaging over random phases involves the use of

$$\langle\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}-i(\omega-\omega')t+i\theta(\mathbf{k},\sigma)-i\theta(\mathbf{k}',\sigma')]\rangle=\delta_{\sigma\sigma'}\delta_{\omega\omega'}\delta^3(\mathbf{k}-\mathbf{k}')\ . \tag{11}$$

With $\int d^3k\to\int d\Omega_k\int dk\,k^2$, (10) can be rewritten as

$$\langle P^{\rm abs}\rangle=\tfrac{1}{2}\mathrm{Re}\left[\frac{e^2}{m}\int d\Omega_k\left[\sum_{\sigma=1}^{2}(\hat{\epsilon}\cdot\hat{\mathbf{x}})^2\right]\times\int dk\,k^2\frac{\hbar\omega}{8\pi^3\epsilon_0}\frac{i\omega}{D^*}\right]\ . \tag{12}$$

Following Boyer[14] we further note that, with the sum over polarizations given by

$$\sum_{\sigma=1}^{2}[\hat{\epsilon}(\mathbf{k},\sigma)\cdot\hat{\mathbf{x}}_i][\hat{\epsilon}(\mathbf{k},\sigma)\cdot\hat{\mathbf{x}}_j]=\delta_{ij}-(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}}_i)(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}}_j)\ , \tag{13}$$

the angular integration in k takes the form

$$\int d\Omega_k\left[\sum_{\sigma=1}^{2}(\hat{\epsilon}\cdot\hat{\mathbf{x}})^2\right]=\int d\Omega_k[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})^2]=\tfrac{8}{3}\pi\ . \tag{14}$$

Substitution of (14) into (12), and a change of variables to $\omega=kc$, then leads to

$$\begin{aligned}\langle P^{\rm abs}\rangle&=\frac{e^2\hbar}{6\pi^2\epsilon_0mc^3}\mathrm{Re}\left[\int_0^\infty\frac{i\omega^4d\omega}{-\omega^2+\omega_0^2+i\Gamma\omega^3}\right]\\&=\frac{e^2\hbar}{6\pi^2\epsilon_0mc^3}\int_0^\infty\frac{\Gamma\omega^7d\omega}{(\omega_0^2-\omega^2)^2+\Gamma^2\omega^6}\ .\end{aligned} \tag{15}$$

Because of the smallness of Γ for a particle with the charge-to-mass ratio of an electron, for this case the integrand in (15) is sharply peaked around $\omega=\omega_0$. We can therefore invoke the standard resonance approximation, extending the limits of integration and replacing ω by ω_0 in all but the difference term. This yields

$$\begin{aligned}\langle P^{\rm abs}\rangle&=\frac{e^2\hbar\omega_0^3}{12\pi\epsilon_0mc^3}\int_{-\infty}^{\infty}\frac{1}{\pi}\frac{(\Gamma\omega_0^2/2)d\omega}{(\omega_0-\omega)^2+(\Gamma\omega_0^2/2)^2}\\&=\frac{e^2\hbar\omega_0^3}{12\pi\epsilon_0mc^3}\ ,\end{aligned} \tag{16}$$

since the (Lorentzian line shape[15]) integral integrates to unity.

In the result (16) we thus have the final expression for the absorption of power from the random background zero-point field by a one-dimensional charged harmonic oscillator.

We now recognize that the Bohr-theory ground-state circular orbit of radius r_0 constitutes a pair of one-dimensional harmonic oscillators in a plane, oscillating in quadrature,

$$x = x_0 \cos\omega_0 t \,,$$
$$y = y_0 \sin\omega_0 t = x_0 \sin\omega_0 t \,, \qquad (17)$$
$$r_0 = (x^2+y^2)^{1/2} = x_0 \,.$$

Therefore, the power absorbed from the background by the electron in circular orbit is double that of (16), or

$$\langle P^{\rm abs} \rangle_{\rm circ} = \frac{e^2 \hbar \omega_0^3}{6\pi\epsilon_0 m c^3} \,. \qquad (18)$$

The power radiated by the electron in circular orbit with acceleration A is given by the standard expression[16]

$$\langle P^{\rm rad} \rangle_{\rm circ} = \frac{e^2 A^2}{6\pi\epsilon_0 c^3} = \frac{e^2 (r_0\omega_0^2)^2}{6\pi\epsilon_0 c^3} = \frac{e^2 r_0^2 \omega_0^4}{6\pi\epsilon_0 c^3} \,. \qquad (19)$$

Under the hypothesis set forward by Boyer,[12] that the ZPF-determined ground-state orbit is set by a balance between radiation emitted due to acceleration of the electron, and radiation absorbed from the zero-point background, we equate (18) and (19) to obtain

$$m\omega_0 r_0^2 = \hbar \,. \qquad (20)$$

We have therefore, within the SED framework, and at the level of Bohr theory, obtained the desired result for the ground state of the hydrogen atom.

DISCUSSION

We have seen that the ground state of the hydrogen atom can be modeled as a ZPF-determined state. The hydrogen atom ground state thus takes its place alongside other treatments, such as the Casimir and van der Waals effects, as an example of a ZPF-determined system successfully analyzed within the SED formulation. The net result of this effort is therefore to increase by one example the number of quantum phenomena that have yielded to the SED approach, while at the same time providing additional insight into the role of ZPF phenomena in determining basic quantum states.[17]

The significance of the derivation is twofold. First, with regard to the analysis technique used, the SED approach used here to solve for the parameters of the ground state of hydrogen as a ZPF-determined state can be taken to constitute an example from the larger program which generally goes under the heading of stochastic mechanics (as outlined, e.g., by Nelson[18]). This larger program attempts to account for quantum fluctuations of particle states on the basis of randomizing physical interactions with a classical background field. Although the nature of the background field in stochastic mechanics is still considered to be an open question, Nelson[19] points out that the electromagnetic field (as used here and in related SED derivations) is a likely candidate, since a necessary requirement is that it be possible to construct a constant with the dimensions of action from the constants of the theory, and this requirement is satisfied in the definition of the fine-structure constant $\alpha = e^2/4\pi\epsilon_0\hbar c$. The level of success obtained here in accounting for the hydrogen ground state on the basis of the SED approach would appear to provide additional supporting evidence for this viewpoint. Further development is required, however, to determine just how far the SED approach can be taken. Of specific interest is whether its success to date is due simply to the correspondence between classical and quantum treatments of linear systems such as nonrelativistic harmonic-oscillator states, or whether a more fundamental role for random classical electromagnetic zero-point radiation is implied. Indeed, for the problem considered here, in the corresponding QED treatment the Heisenberg equations of motion in operator form are formally identical to the equations in this text,[20] and quantum-mechanical ensemble averaging leads to the same results. Thus, the SED treatment and conclusions presented here are reproduced without change in the corresponding QED treatment.

Finally, it is seen that a well-defined, precise quantitative argument can be made that the ground state of the hydrogen atom is defined by a dynamic equilibrium in which collapse of the state is prevented by the presence of zero-point fluctuations of the electromagnetic field. This carries with it the attendant implication that the stability of matter itself is largely mediated by ZPF phenomena in the manner described here, a concept that transcends the usual interpretation of the role and significance of zero-point fluctuations of the vacuum electromagnetic field.

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[13]For completeness we note that other emphasis is possible. In particular, Milonni has argued that the results of field-particle interactions traditionally attributed to ZPF can also be expressed in terms of the radiation reaction of the particles involved, without explicit reference to the ZPF. [See P. W. Milonni, in *Foundations of Radiation Theory and Quantum Electrodynamics* (Ref. 6); Phys. Rev. A **25**, 1315 (1982).] The interrelationship of these two approaches (ZPF, radiation reaction) can be understood in terms of an underlying complementarity expressed by a general fluctuation-dissipation theorem relating this and other pairs of similarly related concepts [see H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951)].
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[15]See, for example, R. H. Pantell and H. E. Puthoff, *Fundamentals of Quantum Electronics* (Wiley, New York, 1969), p. 65.
[16]See, for example, R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1963), Vol. 1, p. 32-2.
[17]An additional step in the hydrogen case remains to be taken, however: namely, to advance from the SED formulation of the circular-orbit ground state of Bohr theory to a satisfactory SED formulation of the spherically symmetric ground state of Schrödinger theory. Although conceptually straightforward as an extension of work to date, a fully satisfactory treatment has yet to be achieved.
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