Introductory Remark

Concerning the following PhD thesis, recently done by Hiroki Sunahata, I would suggest to its perspicacious readers to look carefully at the Appendix C of Ref. [7], ”Contribution to inertial mass by reaction of the vacuum to accelerated motion” by A. Rueda and B. Haisch, Found. Phys., Vol. 28, 1057 (1998). Everything developed there, for the integrations of the stochastic averagings in that Appendix, applies, mutatis mutandis, to the corresponding integrals of the quantum averagings in this thesis. There is an accurate one to one correspondence between the two formalisms. As correctly argued in the thesis this correspondence is known to hold for the electromagnetic field two-point correlation functions which are profusely used in both of these works. Further comments on this correspondence will appear in forthcoming publications.

Alfonso Rueda, Supervisor

Interaction of the Quantum Vacuum with an Accelerated Object and its Contribution to Inertia Reaction Force

BY

Hiroki Sunahata

A Dissertation submitted to the Faculty of Claremont Graduate University and California State University, Long Beach in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Graduate Faculty of Engineering and Industrial Applied Mathematics

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We, the undersigned, certify that we have read this dissertation of Hiroki Sunahata and approve it as adequate in scope and quality for the degree of Doctor of Philosophy.

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Abstract of the Dissertation

Interaction of the Quantum Vacuum with an

Accelerated Object and its Contribution to

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A possible relationship between the zero-point field of the quantum vacuum and the origin of inertia is investigated. The zero-point field (ZPF) is a random, homogeneous, and isotropic electromagnetic field that exists even at the temperature of absolute zero, and its energy density spectrum is Lorentz invariant. Following the approach by Rueda and Haisch (*Found. of Phys. Vol.* **28**, 1057, (1998)), the vacuum expectation value of the ZPF Poynting vector corresponding to the field energy being swept through by the accelerated object per unit time per unit area is evaluated. Here the object is under uniform acceleration, or constant proper acceleration which is known as hyperbolic motion. From this Poynting vector, we can further evaluate the momentum of the background fields the object has swept through as seen from the laboratory frame, and this momentum can then be used to find the force exerted on an accelerated object by the ZPF. This approach had the advantage of avoiding the model dependence used previously by Haisch, Rueda, and Puthoff (*Phys. Rev. A* **49**, 678, (1994)).

Although, in their analysis, Rueda and Haisch used the *classical* stochastic electromagnetic zero-point field, in the present research, the *quantum* formulation for the ZPF is employed using the creation and annihilation operators in the Hilbert space. A relativistic result is reproduced as well by use of the electromagnetic energy-momentum stress tensor which has the Poynting vector components as some of its elements. Similar results are obtained in either approach, and the

force on the accelerating object by the ZPF is found to be proportional and in the opposite direction to the acceleration. Furthermore the proportionality constant turns out to be a scalar quantity with the dimension of mass. Thus the interaction between the accelerated object and the quantum vacuum appears to generate a physical resistance against acceleration, which manifests itself in the form of inertial mass m_i. It has been conjectured by Rueda and collaborators that not only the electromagnetic but other ZPFs such as those of the strong and weak interactions may contribute to the inertial mass.

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Table of Contents

1 Introduction

1.1 Overview

The zero-point field (ZPF) is a random electromagnetic field that exists even at the temperature of absolute zero. The existence of this field first came to be known through the study of the blackbody radiation spectrum early in the twentieth century, and was made more popular with the advance of the quantum theory. Also along with the concept of the zero-point-field, a new classical electromagnetic theory has been proposed [1][2] which includes the zero-point-field as the boundary condition for the Maxwell equations. This new theory has been termed random electrodynamics or stochastic electrodynamics, and it has successfully explained several phenomena which were considered as purely quantum in nature, such as Casimir forces [3] and van der Waals forces [4][5], to name a few.

Moreover, the developments of Stochastic Electrodynamics (SED) in the last decade has expanded its boundary and found new applications. Rueda, Haisch and Puthoff claim that the origin of inertia could be explained, at least in part, as due to the interaction between an accelerated object and the zero-point-field. In their first approach [6], the Lorentz force the ZPF exerts upon the accelerating harmonic oscillator was calculated, and in the second by Rueda and Haisch [7], a more general method was taken by finding out the zero-point-field Poynting vector that an accelerating object of a certain volume V_0 sweeps through. In this theory, this second method will be repeated not in a Stochastic but in a *Quantum Electrodynamics* (QED) approach. It will be shown that the same results follow in QED as well, and that inertia could originate out of the interaction between the accelerated object and the fluctuating quantum vacuum.

We will first review, before we go into the detailed analysis of this thesis, the zero-point-field (ZPF) by itself as well as the theory of Stochastic Electrodynamics.

1.2 Zero-Point-Field

The concept of the zero-point energy first arose in 1911 in Planck's so-called second theory [8] for the blackbody radiation spectrum. He obtained, for the average energy of an oscillator in equilibrium with the radiation field at temperature T,

$$\begin{aligned} U(\nu) &= \frac{1}{2}h\nu\frac{e^{h\nu/2kT}+e^{-h\nu/2kT}}{e^{h\nu/2kT}-e^{-h\nu/2kT}} = \frac{1}{2}h\nu\coth\frac{h\nu}{2kT} \\ &= \frac{1}{2}h\nu\frac{e^{h\nu/kT}+1}{e^{h\nu/kT}-1} = \frac{h\nu}{e^{h\nu/kT}-1}+\frac{1}{2}h\nu, \end{aligned} \tag{1.1}$$

and for the spectral energy density

$$\rho(\nu,T)d\nu = \frac{8\pi\nu^2}{c^3}\left(\frac{h\nu}{e^{h\nu/kT}-1}\right)d\nu. \tag{1.2}$$

It is interesting to note that Planck obtained, in Eq.(1.1), a temperature-independent term $(1/2)h\nu$, suggestive of some residual energy at the temperature of absolute zero for the oscillator energy but failed to obtain in Eq.(1.2) this temperature-independent term for the field, which we now identify as the Zero-Point-Field (ZPF) of the fluctuating vacuum field. In 1913, two years after Planck's "second theory", Einstein and Stern [9] published a paper about the interaction of matter with radiation using a simple dipole oscillator model. In this paper, they remarked that if such an oscillator has a zero-point-energy of $\hbar\omega$ per normal mode, then the equilibrium spectrum of radiation is found to be the Planck spectrum

$$\rho(\omega,T)d\omega = \frac{\hbar\omega^3/\pi^2c^3}{e^{\hbar\omega/kT}-1}d\omega. \tag{1.3}$$

It is clear that Einstein and Stern had attributed the sum of the oscillator ZPF and the field ZPF solely to that of the oscillator. Had they postulated the correct zero-point energy of $(1/2)\hbar\omega$ to both the oscillator and the field, they would have arrived at the correct Planck spectrum with the temperature-independent term,

$$\rho(\nu, T)d\nu = \frac{8\pi\nu^2}{c^3}\left(\frac{h\nu}{e^{h\nu/kT} - 1} + \frac{h\nu}{2}\right)d\nu. \tag{1.4}$$

As this result of Einstein and Stern indicates, there was no concept, at this point, of the zero-point-field. In 1916, Nernst [10] stated that it is impossible to tell the difference between matter and field oscillators if they are in thermal contact to attain statistical equilibrium, and that Planck's equation (1.1) should therefore hold for both matter and field oscillators. This statement of Nernst is generally considered as the birth of the concept of the *zero-point-field*.

In 1924, Mulliken [11] provides us with a direct evidence of the term $(1/2)\hbar\omega$ in the energy levels of the molecular vibrational spectra of boron monoxide. This is regarded as the first evidence of the reality of the zero-point energy, and several months after this Mulliken's discovery, the quantum formalism had begun to be established, in which the concept of the zero-point energy appears so naturally.

1.3 Stochastic Electrodynamics

As a result of the pioneering works in the first half of the twentieth century, mentioned in the previous section, the reality of the zero-point energy and zero-point field had slowly begun to be realized. This opened up in 1960s a new field of physics called Stochastic Electrodynamics (SED). SED is, in Boyer's words [12],

> Lorentz's classical electron theory [13] into which one introduces random electromagnetic radiation (classical zero-point radiation) as the boundary condition giving the homogeneous solution of Maxwell's equations.

The scale of the random radiation is determined with the use of one adjustable parameter, which is chosen in terms of Planck's constant h. This exact form of SED was first proposed by Marshall [1][2], and further developed by Boyer [14], and it has been successful in explaining various quantum phenomena within the framework of traditional classical physics complemented with a classical version of the electromagnetic fluctuations of the vacuum. Some of these successful achievements include: the Casimir effect [3], the Lamb shift [15], the van der Waals forces [16], atomic stability [17], Davies-Unruh effect [18], among many others.[1]

Of particular interest to us in the above is the Davies-Unruh effect, which was discovered in 1975 independently by Davies [20] and Unruh [21] through the study of Hawking radiation of black holes. This Davies-Unruh effect states that, if accelerated through vacuum, an observer finds the surrounding vacuum filled with a heat radiation of temperature $T = \hbar a/2\pi ck$. [2] The meaning of this effect is that an observer under constant acceleration a finds himself/herself as if he/she were immersed *at rest* in a thermal bath of temperature $T = \hbar a/2\pi ck$. This acceleration-dependent Davies-Unruh effect suggests that there exists some unknown structure of the vacuum which reacts only against acceleration. If this hidden structure of the vacuum is activated only if an object is accelerated, then this vacuum might exert a kind of ZPF resistance against accelerated objects, and this could explain the heretofore unexplored origin of inertia. Along this line of thoughts, a series of papers was published by Rueda, Haisch, and Puthoff, and by Rueda and Haisch, in which this ZPF resistive force against acceleration was evaluated using a Planck oscillator model [6], and for a model-independent case [7]. In both cases, they found that the resistive force is proportional in magnitude and in the opposite direction to acceleration. In this thesis, the model-independent case will be studied following Rueda and Haisch's approach, using a *quantum electrodynamical* formulation.

In the next chapter, a mathematical representation of the zero-point field is introduced, along with the coordinate systems we employ. In Chapter 4, detailed calculations of the ZPF Poynting vector (momentum density) that an object sweeps through

[1] For a more detailed history of the developments of SED, refer to de la Pena and Cetto [19], Ch. 4.

[2] Derivation of this Davies-Unruh effect in quantum formulation is given in Appendix F.

under its accelerated motion will be shown and the mathematical form of the electromagnetic vacuum inertial mass component m_i will be determined. A fully covariant approach will be taken in Chapter 5 to obtain the same form for m_i as that obtained in Chapter 4.

2 Zero-Point Radiation

2.1 Zero-Point Field in Classical Random Electrodynamics

The zero-point radiation spectrum has several interesting properties [12]. It is homogeneous and isotropic in every inertial frame [17], Lorentz invariant [2, 22], invariant under an adiabatic compression [10, 23], and invariant under scattering by a dipole oscillator [17] moving with arbitrary constant velocity.

The *classical* electromagnetic zero-point radiation can be written, as a superposition of plane waves [22],

$$\mathbf{E}(\mathbf{R}, t) = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}(\mathbf{k}, \lambda) h_{zp}(\omega) \cos\left[\mathbf{k} \cdot \mathbf{R} - \omega t - \theta(\mathbf{k}, \lambda)\right], \tag{2.1}$$

$$\mathbf{B}(\mathbf{R}, t) = \sum_{\lambda=1}^{2} \int d^3k \left(\hat{k} \times \hat{\epsilon}\right) h_{zp}(\omega) \cos\left[\mathbf{k} \cdot \mathbf{R} - \omega t - \theta(\mathbf{k}, \lambda)\right]. \tag{2.2}$$

Here, the zero-point radiation is expressed in expansion of plane waves and as a sum over two polarization states $\hat{\epsilon}(\mathbf{k}, \lambda)$, which is a function of the propagation vector $\mathbf{k}$ and the polarization index $\lambda = 1, 2$. For each direction of propagation given by $\mathbf{k}$, there exist two mutually orthogonal polarization states $\hat{\epsilon}^1$ and $\hat{\epsilon}^2$, where the superscripts 1 and 2 correspond to the polarization index λ. Hence we have

$$\hat{\epsilon}^l \cdot \hat{\epsilon}^m = \delta_{lm}, \quad l, m = 1, 2, \tag{2.3}$$

$$\hat{\epsilon}^m \cdot \hat{k} = 0, \quad m = 1, 2. \tag{2.4}$$

If we consider the third unit vector $\hat{\epsilon}^3 = \hat{\mathbf{k}} = \mathbf{k}/k$, where $\mathbf{k}$ is the propagation vector, then these three vectors form an orthonormal triad,

$$\sum_{\lambda=1}^{3} \left(\hat{\epsilon}^\lambda\right)_i \left(\hat{\epsilon}^\lambda\right)_j = \sum_{\lambda=1}^{3} \hat{\epsilon}_i^\lambda \hat{\epsilon}_j^\lambda = \hat{\epsilon}_i^1 \hat{\epsilon}_j^1 + \hat{\epsilon}_i^2 \hat{\epsilon}_j^2 + \hat{\epsilon}_i^3 \hat{\epsilon}_j^3 = \delta_{ij}, \tag{2.5}$$

and also

$$\hat{\epsilon}^3 = \hat{k} = \hat{\epsilon}^1 \times \hat{\epsilon}^2 \tag{2.6}$$

and two other relationships of the orthonormal triad can be generated by symmetry: $1 \to 2 \to 3 \to 1$.

Note that, in the above, the polarization components $\hat{\epsilon}_i^\lambda$ are to be understood as scalars. They are the projections of the polarization unit vectors onto the i-axis:

$$\hat{\epsilon}_i^\lambda = \hat{\epsilon}^\lambda \cdot \hat{x}_i, \quad \hat{x}_i = \hat{x}, \hat{y}, \hat{z} \tag{2.7}$$

We also use the same convention with the $\hat{k}$ unit vector, i.e., $\hat{k}_x = \hat{k} \cdot \hat{x}$.

The polarization vectors also satisfy the following relationships:

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i \hat{\epsilon}_j = \delta_{ij} - \hat{k}_i \hat{k}_j \tag{2.8}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i \left(\hat{k} \times \hat{\epsilon}\right)_j = \sum_{k=x,y,z} \varepsilon_{ijk} \hat{k}_k \tag{2.9}$$

$$\sum_{\lambda=1}^{2} \left(\hat{k} \times \hat{\epsilon}\right)_i \left(\hat{k} \times \hat{\epsilon}\right)_j = \delta_{ij} - \hat{k}_i \hat{k}_j \tag{2.10}$$

where ε_{ijk} is a Levi-Civita symbol, and the polarization superscripts λ on the $\hat{\epsilon}$'s are omitted for simplicity. Refer to Appendix A for derivations of the relationships above.

In the expressions (2.1) and (2.2), the random phase $\theta(\mathbf{k}, \lambda)$ is introduced, following Planck [24] and Einstein and Hopf [25], to generate the random, fluctuating nature of the radiation. This $\theta(\mathbf{k}, \lambda)$ is a random variable distributed uniformly in the interval $(0, 2\pi)$ and independently for each wave vector $\mathbf{k}$ and the polarization index λ. Also the spectral function $h_{zp}(\omega)$ is introduced to set the magnitude of the zero-point radiation, which is found in terms of the Planck's constant h as [22]

$$h_{zp}^2(\omega) = \frac{\hbar\omega}{2\pi^2}. \tag{2.11}$$

Plack's constant enters the theory at this point only as a scale factor to attain correspondence between zero temperature random radiation of (classical) stochastic electrodynamics and the vacuum zero point field of quantum electrodynamics. A derivation of this spectral function is also given in Appendix B and it is found that this value for the spectral function is slightly different in the quantum electrodynamical case.

2.2 Zero-Point-Field in Quantum Electrodynamics

In this thesis, a quantum approach is used instead of the classical stochastic approach, to evaluate the vacuum expectation values of the zero-point field. The QED formulation of the zero point electric and magnetic fields are also expressed by the expansion in plane waves as [26, 27]

$$\overline{\mathbf{E}}(\mathbf{r},t) = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}(\mathbf{k},\lambda) H_{zp}(\omega) \left[\alpha(\mathbf{k},\lambda) \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r}) + \alpha^{\dagger}(\mathbf{k},\lambda) \exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})\right], \tag{2.12}$$

$$\overline{\mathbf{B}}(\mathbf{r},t) = \sum_{\lambda=1}^{2} \int d^3k (\hat{k}\times\hat{\epsilon}) H_{zp}(\omega) \left[\alpha(\mathbf{k},\lambda) \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r}) + \alpha^{\dagger}(\mathbf{k},\lambda) \exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})\right]. \tag{2.13}$$

Here the polarization unit vectors $\hat{\epsilon}(\mathbf{k},\lambda)\,(\lambda = 1,2)$ and the wave vector $\mathbf{k}$ are the same as those in the random fields (2.1) and (2.2). The cosine functions in the random electrodynamics formulation are now replaced with the exponential functions and the quantum operators $\alpha(\mathbf{k},\lambda)$ and $\alpha^{\dagger}(\mathbf{k},\lambda)$. These quantum operators are annihilation and creation operators on the Hilbert space and satisfy the commutation rules

$$[\alpha(\mathbf{k},\lambda), \alpha(\mathbf{k}',\lambda')] = \left[\alpha^{\dagger}(\mathbf{k},\lambda), \alpha^{\dagger}(\mathbf{k}',\lambda')\right] = 0 \tag{2.14}$$

$$\left[\alpha(\mathbf{k},\lambda), \alpha^{\dagger}(\mathbf{k}',\lambda')\right] = \delta_{\lambda,\lambda'}\delta^3(\mathbf{k}-\mathbf{k}') \tag{2.15}$$

and have the expectation values,

$$\left\langle 0 \left| \alpha\left(\mathbf{k}, \lambda\right) \alpha\left(\mathbf{k}', \lambda'\right) \right| 0 \right\rangle = \left\langle 0 \left| \alpha^{\dagger}\left(\mathbf{k}, \lambda\right) \alpha^{\dagger}\left(\mathbf{k}', \lambda'\right) \right| 0 \right\rangle = 0 \tag{2.16}$$

$$\left\langle 0 \left| \alpha\left(\mathbf{k}, \lambda\right) \alpha^{\dagger}\left(\mathbf{k}', \lambda'\right) \right| 0 \right\rangle = \delta_{\lambda,\lambda'} \delta^{3}(\mathbf{k} - \mathbf{k}') \tag{2.17}$$

$$\left\langle 0 \left| \alpha^{\dagger}\left(\mathbf{k}, \lambda\right) \alpha\left(\mathbf{k}', \lambda'\right) \right| 0 \right\rangle = 0 \tag{2.18}$$

The overline on E and B in Eq.(2.12) and (2.13) indicates that these fields are now given as operators.

Also the spectral function, introduced to set the scale of the radiations, is expessed as $H_{zp}(\omega)$ using an uppercase H to distinguish this from the classical $h_{zp}(\omega)$. It is found in Appendix B that the value of this $H_{zp}(\omega)$ is

$$H_{zp}^{2}(\omega) = \frac{\hbar\omega}{4\pi^{2}}, \tag{2.19}$$

which is not the same as the classical case $h_{zp}^{2}(\omega) = \hbar\omega/2\pi^{2}$.

3 Correspondence between Random and Quantum Zero-Point Field

In Rueda and Haisch's classical approach, averaged field fluctuations were evaluated using the two-point correlation functions. In the quantum approach used in the present research, however, the expectation values of the vacuum field will be evaluated using the creation and annihilation operators. Although these two approaches are similar in some respects, there also exist several major differences, which was first pointed out by Boyer [26].

In this chapter, following Boyer's analysis, a brief comparison is presented between random electrodynamics and quantum electrodynamics. The connection between the two-point correlation functions in free-field quantum electrodynamics and in random electrodynamics is examined, and it is found that they are in general not equal to each other because of the dependence on the order of the quantum operators. However, if all the products of quantum operators are symmetrized by taking all the permutations of the operator order, then the two theories yield identical results for the correlation functions.

3.1 Two-Point Correlation Function

The electromagnetic field fluctuations may be characterized by the field correlation functions at two different points in space and in time. Therefore, in order to evaluate the correlation in random electrodynamics, averaging over the random phases is taken, and we obtain

$$\left\langle \mathbf{E}_i(\mathbf{r}_1,t_1)\mathbf{E}_j(\mathbf{r}_2,t_2)\right\rangle = \sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3k_1\int d^3k_2\hat{\epsilon}_1\left(\mathbf{k}_1,\lambda_1\right)\hat{\epsilon}_2\left(\mathbf{k}_2,\lambda_2\right)h_{zp}\left(\mathbf{k}_1,\lambda_1\right)h_{zp}\left(\mathbf{k}_2,\lambda_2\right)$$
$$\times\left\langle\cos[\mathbf{k}_1\cdot\mathbf{r}_1-\omega_1t_1+\theta(\mathbf{k}_1,\lambda_1)]\cos[\mathbf{k}_2\cdot\mathbf{r}_2-\omega_2t_2+\theta(\mathbf{k}_2,\lambda_2)]\right\rangle$$
$$=\sum_{\lambda=1}^{2}\int d^3k\hat{\epsilon}_i\left(\mathbf{k},\lambda\right)\hat{\epsilon}_j\left(\mathbf{k},\lambda\right)h_{zp}^2\left(\mathbf{k},\lambda\right)\frac{1}{2}\cos[\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)-\omega(t_1-t_2)]$$
$$=\int d^3k\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)\frac{\hbar\omega}{4\pi^2}\cos[\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)-\omega(t_1-t_2)], \tag{3.1}$$

where the averages

$$\langle\cos\theta(\mathbf{k}_1,\lambda_1)\cos\theta(\mathbf{k}_2,\lambda_2)\rangle = \langle\sin\theta(\mathbf{k}_1,\lambda_1)\sin\theta(\mathbf{k}_2,\lambda_2)\rangle$$
$$=\frac{1}{2}\delta_{\lambda_1\lambda_2}\delta^3\left(\mathbf{k}_1-\mathbf{k}_2\right) \tag{3.2}$$

and

$$\langle\cos\theta(\mathbf{k}_1,\lambda_1)\sin\theta(\mathbf{k}_2,\lambda_2)\rangle = 0 \tag{3.3}$$

were used in the second equality. Also the polarization relation (2.8), i.e.,

$$\sum_{\lambda=1}^{2}\hat{\epsilon}_i\hat{\epsilon}_j = \delta_{ij}-\hat{k}_i\hat{k}_j \tag{3.4}$$

was used in the last equality.

It can be easily shown by similar calculations that we also obtain

$$\left\langle \mathbf{B}_i(\mathbf{r}_1,t_1)\mathbf{B}_j(\mathbf{r}_2,t_2)\right\rangle = \left\langle \mathbf{E}_i(\mathbf{r}_1,t_1)\mathbf{E}_j(\mathbf{r}_2,t_2)\right\rangle \tag{3.5}$$

and

$$\left\langle \mathbf{E}_i(\mathbf{r}_1,t_1)\mathbf{B}_j(\mathbf{r}_2,t_2)\right\rangle = \int d^3k\varepsilon_{ikl}\hat{k}_l\frac{\hbar\omega}{4\pi^2}\cos[\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)-\omega(t_1-t_2)]. \tag{3.6}$$

In quantum electrodynamics, analogous expressions can be obtained as well with the use of the expectation values (2.16)-(2.18), and the polarization equations (2.8)-(2.10). For example, the vacuum expectation value of two electric zero-point field at two different space $\mathbf{R}_1$ and $\mathbf{R}_2$ and two different time t_1 and t_2 would be,

$$\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1) \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2) \right| 0 \right\rangle = \sum_{\lambda_1=1}^{2} \sum_{\lambda_2=1}^{2} \int d^3k_1 \int d^3k_2 \hat{\epsilon}_1 (\mathbf{k}_1, \lambda_1) \, \hat{\epsilon}_2 (\mathbf{k}_2, \lambda_2) \times H_{zp} (\mathbf{k}_1, \lambda_1) \, H_{zp} (\mathbf{k}_2, \lambda_2) \left\langle 0 \left| \left[\alpha(\mathbf{k}_1, \lambda_1) e^{i\Theta_1} + \alpha^\dagger(\mathbf{k}_1, \lambda_1) e^{-i\Theta_1} \right] \right. \right. \times \left. \left. \left[\alpha(\mathbf{k}_2, \lambda_2) e^{i\Theta_2} + \alpha^\dagger(\mathbf{k}_2, \lambda_2) e^{-i\Theta_2} \right] \right| 0 \right\rangle, \quad (3.7)$$

where the form of the ZPF is given by Eq.(2.12), and

$$\Theta_1 = \mathbf{k}_1 \cdot \mathbf{r}_1 - \omega_1 t_1,$$
$$\Theta_2 = \mathbf{k}_2 \cdot \mathbf{r}_2 - \omega_2 t_2. \quad (3.8)$$

Note that we are not allowed to change the order of any operators in the quantum case. Hence the expression above has four terms and each of them has to be evaluated independently. However, we can easily see from the relations Eq.(2.16)-Eq.(2.18), that only one term proportional to $\alpha(\mathbf{k}_1, \lambda_1)\alpha^\dagger(\mathbf{k}_2, \lambda_2)$ is non-vanishing. Therefore, the above equation simplifies to

$$\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1) \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2) \right| 0 \right\rangle = \sum_{\lambda_1=1}^{2} \sum_{\lambda_2=1}^{2} \int d^3k_1 \int d^3k_2 \hat{\epsilon}_1 (\mathbf{k}_1, \lambda_1) \, \hat{\epsilon}_2 (\mathbf{k}_2, \lambda_2) \times H_{zp} (\mathbf{k}_1, \lambda_1) \, H_{zp} (\mathbf{k}_2, \lambda_2) \left\langle 0 \left| \alpha(\mathbf{k}_1, \lambda_1) \alpha^\dagger(\mathbf{k}_2, \lambda_2) \right| 0 \right\rangle \times \exp(-i\omega_1 t_1 + i\mathbf{k}_1 \cdot \mathbf{r}_1) \exp(i\omega_2 t_2 - i\mathbf{k}_2 \cdot \mathbf{r}_2), \quad (3.9)$$

which, with the help of Eq.(2.17), yields the desired result

$$\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1) \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2) \right| 0 \right\rangle = \int d^3k \left(\delta_{ij} - \hat{k}_i \hat{k}_j \right) \frac{\hbar\omega}{4\pi^2} \exp[i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2) - i\omega(t_1 - t_2)] \quad (3.10)$$

In a similar manner, the following two relationships can also be found:

$$\left\langle 0 \left| \overline{\mathbf{B}}_i(\mathbf{r}_1, t_1) \overline{\mathbf{B}}_j(\mathbf{r}_2, t_2) \right| 0 \right\rangle = \left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1) \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2) \right| 0 \right\rangle \tag{3.11}$$

$$\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1) \overline{\mathbf{B}}_j(\mathbf{r}_2, t_2) \right| 0 \right\rangle = \int d^3k \left(\varepsilon_{ijl} \hat{k}_l \right) \frac{\hbar\omega}{4\pi^2} \exp[i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2) - i\omega(t_1 - t_2)] \tag{3.12}$$

3.2 Discrepancies between SED and QED

From the results in the previous section, we see that the expressions for the average do not agree with each other. However, these discrepancies can easily be explained in terms of the operator order. In random electrodynamics, the order of the fields has no significance upon the averaging, i.e.,

$$\left\langle \mathbf{E}_i(\mathbf{r}_1, t_1) \mathbf{E}_j(\mathbf{r}_2, t_2) \right\rangle = \left\langle \mathbf{E}_j(\mathbf{r}_1, t_1) \mathbf{E}_i(\mathbf{r}_2, t_2) \right\rangle . \tag{3.13}$$

On the other hand, in quantum electrodynamics, the operators do not commute and the order does matter,

$$\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1) \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2) \right| 0 \right\rangle \neq \left\langle 0 \left| \overline{\mathbf{E}}_j(\mathbf{r}_1, t_1) \overline{\mathbf{E}}_i(\mathbf{r}_2, t_2) \right| 0 \right\rangle \tag{3.14}$$

In quantum mechanics, physical observables are expressed in terms of *Hermitian* operators. It can be shown that if we symmetrize these operators by taking the every possible permutations and then average the sum, there exists exact agreement between the correlations in random and quantum electrodynamics. To show this correspondence, we first notice that the correlation function (3.1) and the vacuum expectation value (3.10) in quantum electrodynamics have the same form and the only difference is that the cosine function is replaced by the exponentials in QED.

Let us consider the Eq(3.10) and the different order of it in the electric fields,

$$\left\langle 0 \left| \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2)\overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\right| 0 \right\rangle = \int d^3k \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) \frac{\hbar\omega}{4\pi^2} \exp[i\mathbf{k}\cdot(\mathbf{r}_2 - \mathbf{r}_1) - i\omega(t_2 - t_1)]$$
$$= \int d^3k \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) \frac{\hbar\omega}{4\pi^2} \exp[-\{i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2) - i\omega(t_1 - t_2)\}]. \tag{3.15}$$

Note that Eq(3.10) and Eq(3.15) are slightly different in that the exponent has a negative sign in Eq(3.15). Now we add Eq(3.10) and the equation above to obtain

$$\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\overline{\mathbf{E}}_j(\mathbf{r}_2, t_2)\right| 0 \right\rangle + \left\langle 0 \left| \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2)\overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\right| 0 \right\rangle$$
$$= \int d^3k \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) \frac{\hbar\omega}{4\pi^2} \{\exp[i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2) - i\omega(t_1 - t_2)]$$
$$+ \exp[-\{i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2) - i\omega(t_1 - t_2)]\}$$
$$= \int d^3k \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) \frac{\hbar\omega}{4\pi^2} 2\cos[\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2) - i\omega(t_1 - t_2)] \tag{3.16}$$

Notice the presence of the extra factor of two in the last equality, which does not exist in the SED case. This result produces the following relationship,

$$\frac{1}{2}\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\overline{\mathbf{E}}_j(\mathbf{r}_2, t_2) + \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2)\overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\right| 0 \right\rangle$$
$$= \int d^3k \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right) \frac{\hbar\omega}{4\pi^2} \cos[\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2) - i\omega(t_1 - t_2)]$$

and the desired correspondence,

$$\left\langle \mathbf{E}_i(\mathbf{r}_1, t_1)\mathbf{E}_j(\mathbf{r}_2, t_2)\right\rangle = \frac{1}{2}\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\overline{\mathbf{E}}_j(\mathbf{r}_2, t_2) + \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2)\overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\right| 0 \right\rangle$$
$$= \frac{1}{2}\left[\left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\overline{\mathbf{E}}_j(\mathbf{r}_2, t_2)\right| 0 \right\rangle + \left\langle 0 \left| \overline{\mathbf{E}}_j(\mathbf{r}_2, t_2)\overline{\mathbf{E}}_i(\mathbf{r}_1, t_1)\right| 0 \right\rangle\right] \tag{3.17}$$

is obtained. It can also be easily shown that this agreement between the correlation function in random electrodynamics and the vacuum expectation value in quantum electrodynamics also holds for the other two-point functions (3.11) and (3.12) as well. These discrepancies between SED and QED, stemming from the non-commutivity of the quantum operators keep arising in all orders. However, regardless of the order, if we construct the symmetrized operators, the correspondence between two theories would be achieved [26].

3.3 Heisenberg Picture and Schroedinger Picture

In a formulation of a system in quantum mechanics, time evolution can be treated in two different manners: we can either absorb the time evolution in the state vector $|\Psi(t)\rangle$ and treat the operator as constant in time, or let the state vector to be time constant and treat the operator as a time dependent quantity, $A = A(t)$. The former is called the *Schroedinger picture* and the latter the *Heisenberg picture*. In our research, we are obviously adopting the Heisenberg picture, for our state vector $|0\rangle$ is always fixed in time and the time dependence is absorbed in the operators.

It is known that the difference between these two formulations is just the way in which time evolution is handled and they are in most cases equivalent otherwise. [3] The results and predictions of quantum mechanics are not affected by the choice of the formulation. Therefore, the correspondence between the SED and QED also remains unaffected by our choice of the Heisenberg picture.

Regarding the Heisenberg picture and its agreement with random electrodynamics, Milonni[4] remarks the following:

> The equations of motion are formally the same in the two theories. The final step in the derivation involves a term bilinear in the zero-point electric field. In the quantum electrodynamics case we require an expectation

[3] A possibility of discrepancy between the Schrodinger and the Heisenberg picture has been pointed out by A. J. Faria et al.[28]. They claim that the effects of the zero-point field may be counted twice in the Schrodinger treatment of the oscillator.

[4] P.W. Milonni, Physics Reports **25**, No.1 (1976) pp. 1-81.

> value of this term over the vacuum field state. In random electrodynamics we require an average of formally the same term over the random phases of the zero-point field. The two types of ensemble average yield the same answer, and therefore the same result for the van der Waals interaction. From this view point, we might even contend that the principal merit in Boyer's derivation is the treatment of the problem in the Heisenberg picture, with a consequent ease of physical interpretation.

Although this comment was made on the derivation of the van der Waals forces, our research involves essentially the same procedure: the expectation values of the electromagnetic fields over the vacuum state in quantum electrodynamics, and the averages of the fields over the random phases of the zero-point field in random electrodynamics. The strong similarity between the Heisenberg picture treatment and random electrodynamics is equally valid in our research as well.

4 Evaluation of the ZPF Poynting Vector

4.1 Overview

In this chapter, the Poynting vector $\mathbf{S}_*^{zp}$ of the zero-point field that an object sweeps through under its constantly accelerated motion is evaluated in the laboratory inertial frame. The object has a proper volume V_0 in its own rest frame and is accelerated along the positive x-direction, that is, $\mathbf{a} = a\hat{x}$. Since the Poynting vector is physically a momentum flux, we can also find out its momentum density $\mathbf{g}_*^{zp}$ by dividing the Poynting vector by c^2, i.e., $\mathbf{g}_*^{zp} = \mathbf{S}_*^{zp}/c^2$. Since the ZPF spectrum is Lorentz-invariant, this momentum density is a time-independent constant under constant velocity. However, under the constant *acceleration* we consider in this research, this momentum density will become a function of time. Therefore, once we find out this momentum density, the force that the ZPF exerts upon the accelerated object can be determined as well by taking the time derivative of the momentum. It will be shown that this ZPF resistive force is directly proportional to the acceleration and directed against the acceleration.

Consider an inertial observer at rest in the laboratory frame I_*. This observer will find that the ZPF is isotropically distributed around himself, and that the ZPF Poynting vector $\mathbf{S}_*^{zp}$ is zero, for there is no flux of ZPF in this situation. Now consider an object moving with constant velocity along the x-axis, $\mathbf{v} = v_x\hat{x}$. Since the ZPF spectrum is Lorentz-invariant, both the stationary I_*-observer and the observer comoving with the object will see the ZPF isotropically distributed around themselves. However, the I_*-observer will not find the ZPF of the other observer comoving with the object isotropically distributed around himself, since the Doppler effects shift the wave vector $\mathbf{k}$ depending on the velocity of the object. This of course is true for the ZPF of the I_*-observer as seen from the moving observer as well. In this situation where the object is moving with constant velocity, the I_*-observer will find that the object carries a momentum $\mathbf{p}_* = \gamma m_0 \mathbf{v}$, and that both the ZPF Poynting vector $\mathbf{S}_*^{zp}$ and its corresponding momentum density $\mathbf{g}_*^{zp} = \mathbf{S}_*^{zp}/c^2$ are non-zero, which remain constant independent of time. Moreover, if the object is under hyperbolic motion (constantly accelerated motion), then the ZPF spectrum is not time independent any more and the ZPF momentum seen from the I_*-observer view point becomes a function of time (Eq. (4.54)), and consequently the time derivative of the momentum becomes nonvanishing (Eq. (4.55)).

The discussions above can also be explained in the same manner using the concept of the k-sphere.[5] In evaluating the ZPF Poynting vector, we perform integrations in k-space up to the cut-off radius $k_c = \omega_c/c$ centered at the observation point, with the cut-off frequency ω_c associated with the ZPF spectral distribution. Every inertial observer has his own k-sphere spherically symmetrically distributed around himself. In the case of the constant velocity discussed above, the observer comoving with the object will find that the object is at the center of his own k-sphere, and the object appears to carry no mechanical or ZPF momenta whatsoever. However, this situation would become quite different from the point of view of the I_*-observer, since the object is not located at the center of his own k-sphere. From the I_*-observer's view, the object appears to carry both mechanical momentum $\mathbf{p}_* = \gamma m_0 \mathbf{v}$ and the ZPF momentum density $\mathbf{g}_*^{zp}$, which are both constants independent of time. As soon as this object get accelerated by an external agent, both the ZPF momentum density and the corresponding ZPF momentum $\mathbf{p}_*^{zp} = V_* \mathbf{g}_*^{zp}$ as seen from the I_*-observer become acceleration dependent functions of time, which, later in this chapter, we find to be

$$\mathbf{g}_*^{zp}(\tau) = \frac{\mathbf{S}_*^{zp}(\tau)}{c^2} = -\hat{x}\frac{1}{4\pi c}\frac{8\pi}{3}\sinh\left(\frac{2a\tau}{c}\right)\int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega, \tag{4.1}$$

and

$$\mathbf{p}_*^{zp}(\tau) = V_* \mathbf{g}_*^{zp} = \frac{V_0}{\gamma_\tau}\mathbf{g}_*^{zp}(\tau) = -\hat{x}\frac{4V_0}{3c}\beta_\tau\gamma_\tau \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \tag{4.2}$$

[5] For more detailed explanations of the k-sphere, see A. Rueda and B. Haisch, Found. Phys. 28, 1057, 1998, especially Appendix C.

Since this ZPF momentum density is a function of time, we can find the time derivatives of this function $\mathbf{f}_*^{zp} = d\mathbf{p}_*^{zp}/dt_*$, and we interpret that this is the force that the ZPF exerts upon the accelerated objects.

4.2 Constant Proper Acceleration (Hyperbolic Motion)

As a basis of our analysis, we employ the following system of reference frames [18, 7]: an inertial laboratory frame I_*, the accelerated frame S in which the object is placed at rest at the point $(c^2/a, 0, 0)$, and a set of instantaneous inertial frames I_τ defined at each of the object's proper time τ. The accelerated frame S comoving with the object is set to coincide with the lab frame I_* when $t_* = \tau = 0$.

The object is at rest at the point $(c^2/a, 0, 0)$ in the accelerated frame S, and the acceleration is in the positive x-direction. This acceleration of the $(c^2/a, 0, 0)$ point in S as seen from the instantaneous comoving frame I_τ becomes $\mathbf{a}$. This is a so-called hyperbolic motion [29, 30] since the world line of the object under this fixed acceleration a with respect to the instantaneous rest frame I_τ traces a hyperbola in a spacetime diagram (see Figure 1 below) and hyperbolic functions enter into the temporal description of the motion.

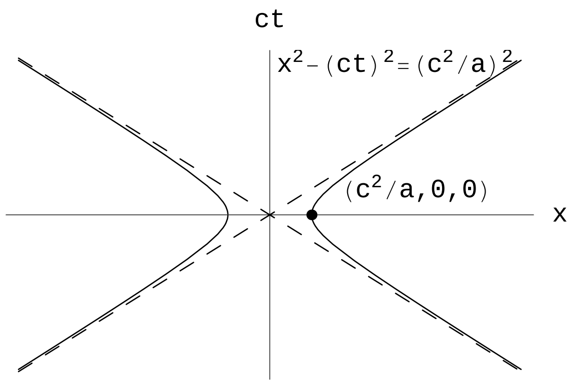

Figure 1: Hyperbolic Motion

In this system of hyperbolic motion, the object's position and time in I_* are expressed in terms of the proper time τ as

$$t_* = \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right), \tag{4.3}$$

$$x_* = \mathbf{R}_*(\tau) \cdot \hat{x} = \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right), \tag{4.4}$$

$$y_* = 0, \tag{4.5}$$

$$z_* = 0, \tag{4.6}$$

where $\mathbf{R}_*$ is the object's coordinates as seen from the I_* lab reference frame, which is chosen to be $(c^2/a, 0, 0)$ in both I_* and S frames when $t = \tau = 0$. Note that $x_*^2 - (ct_*)^2 = c^4/a^2$, which is an equation for a hyperbola.

Similarly, the magnitude of the velocity $u_*(\tau)$ with respect to I_* is given by

$$\beta_\tau = \frac{u_*(\tau)}{c} = \frac{1}{c}\frac{dx_*}{dt_*} = \frac{1}{c}\frac{dx_*/d\tau}{dt_*/d\tau} = \tanh\left(\frac{a\tau}{c}\right), \tag{4.7}$$

where we introduced the normalized velocity β_τ. We also use the corresponding γ-factor, namely,

$$\gamma_\tau = \frac{1}{\sqrt{1-\beta_\tau^2}} = \frac{1}{sech(a\tau/c)} = \cosh\left(\frac{a\tau}{c}\right). \tag{4.8}$$

Both (4.7) and (4.8) are hyperbolic functions.

4.3 Transformation of the Fields

The ZPF in the laboratory frame I_* is given as an expansion in plane waves by

$$\begin{aligned} \mathbf{E}_*^{zp}(\mathbf{R}_*, t_*) = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}\,(\mathbf{k},\lambda)\, H_{zp}(\omega) \\ \times \left\{\alpha\,(\mathbf{k},\lambda) \exp\left[i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)\right] + \alpha^\dagger\,(\mathbf{k},\lambda) \exp\left[-i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)\right]\right\}, \end{aligned} \tag{4.9}$$

and

$$\mathbf{B}_*^{zp}(\mathbf{R}_*, t_*) = \sum_{\lambda=1}^{2} \int d^3k \left(\hat{k} \times \hat{\epsilon}\right) H_{zp}(\omega)$$
$$\times \left\{\alpha\left(\mathbf{k}, \lambda\right) \exp\left[i(\mathbf{k} \cdot \mathbf{R}_* - \omega t_*)\right] + \alpha^\dagger\left(\mathbf{k}, \lambda\right) \exp\left[-i(\mathbf{k} \cdot \mathbf{R}_* - \omega t_*)\right]\right\}, \quad (4.10)$$

where $\mathbf{R}_*$ and t_* are the space and time coordinates of the observation point in I_*. The polarization unit vectors $\hat{\epsilon}\left(\mathbf{k}, \lambda\right)$ $(\lambda = 1, 2)$ are orthogonal to each other and to the wave vector $\mathbf{k}$, and the function $H_{zp}(\omega)$ is defined in such a way that it corresponds to the electromagnetic energy per normal mode at frequency ω, that is,

$$H_{zp}^2(\omega) = \frac{\hbar\omega}{4\pi^2}. \quad (4.11)$$

Since this I_* lab frame is the ultimate reference frame where all the physical quantities of the accelerating object in I_τ frame is to be evaluated, in order to obtain the ZPF that the object is subjected to in the accelerated frame, we transform these fields from the inertial frame I_* to the corresponding I_τ frame using the standard Lorentz-transformation of the fields, namely,

$$\begin{aligned} E_1' &= E_1 & B_1' &= B_1 \\ E_2' &= \gamma(E_2 - \beta B_3) & B_2' &= \gamma(B_2 + \beta E_3) \\ E_3' &= \gamma(E_3 + \beta B_2) & B_3' &= \gamma(B_3 - \beta E_2) \end{aligned} \quad (4.12)$$

and obtain

$$\mathbf{E}_{\tau}^{zp}(0,\tau) = \sum_{\lambda=1}^{2}\int d^3k \left\{\hat{x}\hat{\epsilon}_x + \hat{y}\gamma_\tau[\hat{\epsilon}_y - \beta_\tau(\hat{k}\times\hat{\epsilon})_z] + \hat{z}\gamma_\tau[\hat{\epsilon}_z + \beta_\tau(\hat{k}\times\hat{\epsilon})_y]\right\} H_{zp}(\omega)$$
$$\times\left\{\alpha\,(\mathbf{k},\lambda)\exp\left[i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)\right] + \alpha^{\dagger}\,(\mathbf{k},\lambda)\exp\left[-i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)\right]\right\}, \quad (4.13)$$
$$\mathbf{B}_{\tau}^{zp}(0,\tau) = \sum_{\lambda=1}^{2}\int d^3k \left\{\hat{x}(\hat{k}\times\hat{\epsilon})_x + \hat{y}\gamma_\tau[(\hat{k}\times\hat{\epsilon})_y + \beta_\tau\hat{\epsilon}_z] + \hat{z}\gamma_\tau[(\hat{k}\times\hat{\epsilon})_z - \beta_\tau\hat{\epsilon}_y]\right\}$$
$$\times H_{zp}(\omega)\left\{\alpha\,(\mathbf{k},\lambda)\exp\left[i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)\right] + \alpha^{\dagger}\,(\mathbf{k},\lambda)\exp\left[-i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)\right]\right\}. \quad (4.14)$$

The arguments of the fields are taken as zero but it actually means the I_τ spatial point $(c^2/a, 0, 0)$.

We assume that the object is subject to a constant acceleration, i.e., a hyperbolic motion. Then as seen before (4.3-4.8), the velocity in the comoving frame I_τ with respect to I_* and the space and time coordinates of the object in I_τ are given by

$$\beta_\tau = u_*(\tau)/c = \tanh(a\tau/c), \quad (4.15)$$
$$\gamma_\tau = \frac{1}{\sqrt{1-\beta_\tau^2}} = \frac{1}{sech(a\tau/c)} = \cosh(a\tau/c), \quad (4.16)$$
$$\mathbf{R}_*(\tau)\cdot\hat{x} = (c^2/a)\cosh(a\tau/c), \quad (4.17)$$
$$t_* = (c/a)\sinh(a\tau/c). \quad (4.18)$$

Putting these into the above expressions for the fields (4.13) and (4.14), we obtain,

$$\begin{aligned}
\mathbf{E}_{\tau}^{zp}(0,\tau) = \sum_{\lambda=1}^{2} \int d^3k \Big\{ & \hat{x}\hat{\epsilon}_x + \hat{y}\cosh\left(\frac{a\tau}{c}\right)\left[\hat{\epsilon}_y - \tanh\left(\frac{a\tau}{c}\right)(\hat{k}\times\hat{\epsilon})_z\right] \\
& + \hat{z}\cosh\left(\frac{a\tau}{c}\right)\left[\hat{\epsilon}_z + \tanh\left(\frac{a\tau}{c}\right)(\hat{k}\times\hat{\epsilon})_y\right]\Big\} H_{zp}(\omega) \\
& \times \left\{\alpha\left(\mathbf{k},\lambda\right)\exp\left[i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\right. \\
& \left. + \alpha^{\dagger}\left(\mathbf{k},\lambda\right)\exp\left[-i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\right\} \qquad (4.19) \\
\mathbf{B}_{\tau}^{zp}(0,\tau) = \sum_{\lambda=1}^{2} \int d^3k \Big\{ & \hat{x}(\hat{k}\times\hat{\epsilon})_x + \hat{y}\cosh\left(\frac{a\tau}{c}\right)\left[(\hat{k}\times\hat{\epsilon})_y + \tanh\left(\frac{a\tau}{c}\right)\hat{\epsilon}_z\right] \\
& + \hat{z}\cosh\left(\frac{a\tau}{c}\right)\left[(\hat{k}\times\hat{\epsilon})_z - \tanh\left(\frac{a\tau}{c}\right)\hat{\epsilon}_y\right]\Big\} H_{zp}(\omega) \\
& \times \left\{\alpha\left(\mathbf{k},\lambda\right)\exp\left[i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\right. \\
& \left. + \alpha^{\dagger}\left(\mathbf{k},\lambda\right)\exp\left[-i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\right\} \qquad (4.20)
\end{aligned}$$

where the quantum operators $\alpha\left(\mathbf{k},\lambda\right)$ and $\alpha^{\dagger}\left(\mathbf{k},\lambda\right)$ are the annihilation and creation operators on the Hilbert space respectively, which satisfy the commutation rules

$$\left[\alpha\left(\mathbf{k},\lambda\right),\alpha\left(\mathbf{k}',\lambda'\right)\right] = \left[\alpha^{\dagger}\left(\mathbf{k},\lambda\right),\alpha^{\dagger}\left(\mathbf{k}',\lambda'\right)\right] = 0 \qquad (4.21)$$

$$\left[\alpha\left(\mathbf{k},\lambda\right),\alpha^{\dagger}\left(\mathbf{k}',\lambda'\right)\right] = \delta_{\lambda,\lambda'}\delta^3(\mathbf{k}-\mathbf{k}') \qquad (4.22)$$

and have the expectation values,

$$\left\langle 0\left|\alpha\left(\mathbf{k},\lambda\right)\alpha\left(\mathbf{k}',\lambda'\right)\right|0\right\rangle = \left\langle 0\left|\alpha^{\dagger}\left(\mathbf{k},\lambda\right)\alpha^{\dagger}\left(\mathbf{k}',\lambda'\right)\right|0\right\rangle = 0 \qquad (4.23)$$

$$\left\langle 0\left|\alpha\left(\mathbf{k},\lambda\right)\alpha^{\dagger}\left(\mathbf{k}',\lambda'\right)\right|0\right\rangle = \delta_{\lambda,\lambda'}\delta^3(\mathbf{k}-\mathbf{k}') \qquad (4.24)$$

$$\left\langle 0\left|\alpha^{\dagger}\left(\mathbf{k},\lambda\right)\alpha\left(\mathbf{k}',\lambda'\right)\right|0\right\rangle = 0 \qquad (4.25)$$

Notice here that the order of the quantum operator affects the result as mentioned in the earlier chapter. This problem does not arise in the classical random variable cases.

The expressions for the fields above, (4.19) and (4.20) are the ZPF as instantaneously viewed from the object fixed to the point $(c^2/a, 0, 0)$ of S that is performing the hyperbolic motion.

4.4 Evaluation of the Poynting vector components

We now evaluate the ZPF Poynting vector corresponding to the radiation being swept through by the accelerated object as seen from the observer at rest at $(c^2/a, 0, 0)$ of I_*, using the expression for the fields obtained in the previous section, that is,

$$
\begin{aligned}
\mathbf{E}_\tau^{zp}(0,\tau) = \sum_{\lambda=1}^{2} \int d^3k \Bigg\{ & \hat{x}\hat{\epsilon}_x + \hat{y}\cosh\left(\frac{a\tau}{c}\right)\left[\hat{\epsilon}_y - \tanh\left(\frac{a\tau}{c}\right)(\hat{k}\times\hat{\epsilon})_z\right] \\
& + \hat{z}\cosh\left(\frac{a\tau}{c}\right)\left[\hat{\epsilon}_z + \tanh\left(\frac{a\tau}{c}\right)(\hat{k}\times\hat{\epsilon})_y\right]\Bigg\} H_{zp}(\omega) \\
& \times \Bigg\{ \alpha(\mathbf{k},\lambda)\exp\left[i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right] \\
& + \alpha^\dagger(\mathbf{k},\lambda)\exp\left[-i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\Bigg\}
\end{aligned}
\tag{4.26}
$$

$$
\begin{aligned}
\mathbf{B}_\tau^{zp}(0,\tau) = \sum_{\lambda=1}^{2} \int d^3k \Bigg\{ & \hat{x}(\hat{k}\times\hat{\epsilon})_x + \hat{y}\cosh\left(\frac{a\tau}{c}\right)\left[(\hat{k}\times\hat{\epsilon})_y + \tanh\left(\frac{a\tau}{c}\right)\hat{\epsilon}_z\right] \\
& + \hat{z}\cosh\left(\frac{a\tau}{c}\right)\left[(\hat{k}\times\hat{\epsilon})_z - \tanh\left(\frac{a\tau}{c}\right)\hat{\epsilon}_y\right]\Bigg\} H_{zp}(\omega) \\
& \times \Bigg\{ \alpha(\mathbf{k},\lambda)\exp\left[i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right] \\
& + \alpha^\dagger(\mathbf{k},\lambda)\exp\left[-i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\Bigg\}
\end{aligned}
\tag{4.27}
$$

In all of the evaluations of the vacuum expectation values for the electric and magnetic components of the ZPF, $\left\langle 0 \left| E_i B_j \right| 0 \right\rangle$, $i, j = x, y, z$, to follow in this section, we need to evaluate the following quantity,

$$
\left\langle 0 \left| \left\{ \alpha(\mathbf{k},\lambda)\, e^{i\Theta} + \alpha^\dagger(\mathbf{k},\lambda)\, e^{-i\Theta} \right\} \times \left\{ \alpha(\mathbf{k}',\lambda')\, e^{i\Theta'} + \alpha^\dagger(\mathbf{k}',\lambda')\, e^{-i\Theta'} \right\} \right| 0 \right\rangle
\tag{4.28}
$$

where

$$\Theta = k_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right) \tag{4.29}$$

and

$$\Theta' = k'_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega' \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right) \tag{4.30}$$

In order to evaluate this quantity, we make use of the expectation value relationships for the quantum creation and annihilation operators (4.23)-(4.25), and it is found that only terms proportional to $\left\langle 0 \left| \alpha\left(\mathbf{k}, \lambda\right) \alpha^{\dagger}\left(\mathbf{k}', \lambda'\right) \right| 0 \right\rangle$ remain and all the other terms vanish. Therefore, we have

$$\begin{aligned} \left\langle 0 \left| E_i(\mathbf{k}, \lambda) B_j(\mathbf{k}', \lambda') \right| 0 \right\rangle &\propto e^{i\Theta} e^{-i\Theta'} \times \left\langle 0 \left| \alpha\left(\mathbf{k}, \lambda\right) \alpha^{\dagger}\left(\mathbf{k}', \lambda'\right) \right| 0 \right\rangle \\ &\propto \delta_{\lambda,\lambda'} \delta^3(\mathbf{k} - \mathbf{k}') e^{i\Theta(\mathbf{k})} e^{-i\Theta'(\mathbf{k}')} \end{aligned} \tag{4.31}$$

where

$$\begin{aligned} \Theta(\mathbf{k}) &= k_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right), \\ \Theta'(\mathbf{k}') &= k'_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega' \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right). \end{aligned}$$

As explained in the previous chapter, in order to assure the correspondence between the random electrodynamics and the quantum electrodynamics results, it is required to construct the symmetrized operators in the QED case by taking every possible permutations of the field operators, that is,

$$\left\langle \mathbf{E}_i(\mathbf{r}_1, t_1) \mathbf{B}_j(\mathbf{r}_2, t_2) \right\rangle = \frac{1}{2} \left\langle 0 \left| \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1) \overline{\mathbf{B}}_j(\mathbf{r}_2, t_2) + \overline{\mathbf{B}}_j(\mathbf{r}_2, t_2) \overline{\mathbf{E}}_i(\mathbf{r}_1, t_1) \right| 0 \right\rangle. \tag{4.32}$$

However, due to the symmetrical form of the zero-point field with respect to the quantum operators, i.e.,

$$\mathbf{E}_i(\mathbf{k},\lambda) \propto \left\{\alpha\,(\mathbf{k},\lambda)\,e^{i\Theta} + \alpha^{\dagger}\,(\mathbf{k},\lambda)\,e^{-i\Theta}\right\}, \tag{4.33}$$

$$\mathbf{B}_j(\mathbf{k}',\lambda') \propto \left\{\alpha\,(\mathbf{k}',\lambda')\,e^{i\Theta'} + \alpha^{\dagger}\,(\mathbf{k}',\lambda')\,e^{-i\Theta'}\right\}, \tag{4.34}$$

and the expectation value relationship (4.23)-(4.25), it is clear that

$$\left\langle 0\left|\overline{\mathbf{E}}_i(\mathbf{k},\lambda)\overline{\mathbf{B}}_j(\mathbf{k}',\lambda')\right|0\right\rangle = \left\langle 0\left|\overline{\mathbf{B}}_j(\mathbf{k}',\lambda')\overline{\mathbf{E}}_i(\mathbf{k},\lambda)\right|0\right\rangle, \tag{4.35}$$

which gives us a simple relation between SED and QED,

$$\left\langle \mathbf{E}_i(\mathbf{k},\lambda)\mathbf{B}_j(\mathbf{k}',\lambda')\right\rangle = \left\langle 0\left|\overline{\mathbf{E}}_i(\mathbf{k},\lambda)\overline{\mathbf{B}}_j(\mathbf{k}',\lambda')\right|0\right\rangle, \tag{4.36}$$

which always holds in the cases of our interest.

Now, we proceed with the explicit evaluations of the ZPF expectation values. Let $\mathbf{S}^{zp}$ denote the ZPF Poynting vector. Then $\mathbf{S}^{zp}_*$, the ZPF Poynting vector that enters the body of the accelerating object in the instantaneous comoving frame I_τ, evaluated from the laboratory inertial frame I_*, is given by

$$\begin{aligned}
\mathbf{S}^{zp}_* &= \frac{c}{4\pi}\left\langle 0\left|E^{zp}_\tau \times B^{zp}_\tau\right|0\right\rangle_* \\
&= \frac{c}{4\pi}\left\{\hat{x}\left\langle 0\left|E_yB_z - E_zB_y\right|0\right\rangle + \hat{y}\left\langle 0\left|E_zB_x - E_xB_z\right|0\right\rangle\right. \\
&\qquad \left. +\ \hat{z}\left\langle 0\left|E_xB_y - E_yB_x\right|0\right\rangle\right\}.
\end{aligned} \tag{4.37}$$

It turns out that only the x-component of the ZPF Poynting vector is non-vanishing and the other components are zero. The detailed calculations of each component of this Poynting vector is shown in Appendix C, and only a brief summary of this evaluation will be shown below.

In order to evaluate the vacuum expectation value for a component of the ZPF

Poynting vector, e.g., $\langle 0 |E_x B_x| 0\rangle$, the x-component of the zero-point electric field operators (4.26), that is,

$$\begin{aligned}\mathbf{E}^{zp}_{\tau x}(0,\tau) = \sum_{\lambda=1}^{2}\int d^3k\,\{\hat{x}\hat{\epsilon}_x\}\,H_{zp}(\omega) \\ \times\left\{\alpha\,(\mathbf{k},\lambda)\exp\left[i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right)-\omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\right. \\ \left.+\ \alpha^{\dagger}\,(\mathbf{k},\lambda)\exp\left[-i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right)-\omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\right\}\end{aligned} \tag{4.38}$$

and the y-component of the zero-point magnetic field operators (4.27), i.e.,

$$\begin{aligned}\mathbf{B}^{zp}_{\tau x}(0,\tau) = \sum_{\lambda=1}^{2}\int d^3k\left\{\hat{x}(\hat{k}\times\hat{\epsilon})_x H_{zp}(\omega)\right\} \\ \times\left\{\alpha\,(\mathbf{k},\lambda)\exp\left[i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right)-\omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\right. \\ \left.+\ \alpha^{\dagger}\,(\mathbf{k},\lambda)\exp\left[-i\left(k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right)-\omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right)\right)\right]\right\}\end{aligned} \tag{4.39}$$

are multiplied together and the following expression is obtained:

$$\begin{aligned}\langle 0 |E_x B_x| 0\rangle = \sum_{\lambda=1}^{2}\sum_{\lambda'=1}^{2}\int d^3k\int d^3k'\hat{\epsilon}_x(\hat{k}'\times\hat{\epsilon}')_x H^2_{zp}(\omega) \\ \left\langle 0\left|\left\{\alpha\,(\mathbf{k},\lambda)\exp[i\Theta]+\alpha^{\dagger}\,(\mathbf{k},\lambda)\exp[-i\Theta]\right\}\right.\right. \\ \left.\left.\times\ \left\{\alpha\,(\mathbf{k}',\lambda')\exp[i\Theta']+\alpha^{\dagger}\,(\mathbf{k}',\lambda')\exp[-i\Theta']\right\}\right|0\right\rangle,\end{aligned} \tag{4.40}$$

where

$$\Theta = k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right)-\omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right) \tag{4.41}$$

and

$$\Theta' = k'_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right)-\omega'\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right). \tag{4.42}$$

The above expression has four terms, but only the term that is proportional to $\left\langle 0\left|\alpha\,(\mathbf{k},\lambda)\,\alpha^{\dagger}\,(\mathbf{k}',\lambda')\right|0\right\rangle$ remains as in (4.24), and the above expression is simplified to

$$\langle 0 | E_x B_x | 0 \rangle = \sum_{\lambda=1}^{2} \sum_{\lambda'=1}^{2} \int d^3k \int d^3k' \hat{\epsilon}_x (\hat{k}' \times \hat{\epsilon}')_x H_{zp}^2(\omega)$$
$$\times \left\langle 0 \left| \alpha(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') \right| 0 \right\rangle \exp[i\Theta(\mathbf{k})] \exp[-i\Theta'(\mathbf{k}')], \tag{4.43}$$

with the use of the expectation values

$$\left\langle 0 \left| \alpha(\mathbf{k}, \lambda) \alpha(\mathbf{k}', \lambda') \right| 0 \right\rangle = \left\langle 0 \left| \alpha^\dagger(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') \right| 0 \right\rangle = 0 \tag{4.44}$$
$$\left\langle 0 \left| \alpha(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') \right| 0 \right\rangle = \delta_{\lambda,\lambda'} \delta^3(\mathbf{k} - \mathbf{k}') \tag{4.45}$$
$$\left\langle 0 \left| \alpha^\dagger(\mathbf{k}, \lambda) \alpha(\mathbf{k}', \lambda') \right| 0 \right\rangle = 0. \tag{4.46}$$

Since the term in the second line in (4.43) is $\delta_{\lambda,\lambda'} \delta^3(\mathbf{k} - \mathbf{k}') \exp[i\Theta(\mathbf{k})] \exp[-i\Theta'(\mathbf{k}')]$, Eq.(4.43) becomes

$$\langle 0 | E_x B_x | 0 \rangle = \sum_{\lambda=1}^{2} \sum_{\lambda'=1}^{2} \int d^3k \int d^3k' \hat{\epsilon}_x (\hat{k}' \times \hat{\epsilon}')_x H_{zp}^2(\omega)$$
$$\times \delta_{\lambda,\lambda'} \delta^3(\mathbf{k} - \mathbf{k}') \exp[i\Theta(\mathbf{k})] \exp[-i\Theta'(\mathbf{k}')], \tag{4.47}$$

which, after one integration over the k-sphere, reduces to

$$\langle 0 | E_x B_x | 0 \rangle = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_x H_{zp}^2(\omega). \tag{4.48}$$

With a use of the polarization formula (A.7) derived in Appendix A, we find that

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_x = \sum_{k=x,y,z} \varepsilon_{iik} \hat{k}_k = 0, \tag{4.49}$$

and after substituting this result into the equation above, it is concluded that $\langle 0 | E_x B_x | 0 \rangle = 0$.

The remaining eight components of the Poynting vector can also be evaluated in a similar manner (for detailed calculations, refer to Appendix C), and it is found that only the following two terms remain non-vanishing, each of which has the same magnitude and in the opposite direction to each other:

$$\left\langle 0 \left| E_y B_z \right| 0 \right\rangle = -\frac{4\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega, \tag{4.50}$$

and

$$\left\langle 0 \left| E_z B_y \right| 0 \right\rangle = \frac{4\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \tag{4.51}$$

With the results above, the Poynting vector $\mathbf{S}_*^{zp}$ (4.37) becomes

$$\begin{aligned} \mathbf{S}_*^{zp} &= \hat{x} \frac{c}{4\pi} \left\langle 0 \left| E_\tau^{zp}(0,\tau) \times B_\tau^{zp}(0,\tau) \right| 0 \right\rangle_x \\ &= -\hat{x} \frac{c}{4\pi} \frac{8\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \end{aligned} \tag{4.52}$$

This represents the energy flux, i.e., the ZPF energy that enter the uniformly accelerating object's body per unit area per unit time as seen from the observer at rest in the inertial laboratory frame I_*.

4.5 Derivation of the Inertial Mass

In the previous section and in Appendix C, all of the Poynting vector components $\left\langle 0 \left| E_i B_j \right| 0 \right\rangle$ where $i, j = x, y, z$ were evaluated and it turned out that all the components vanish except two, and the ZPF Poynting vector turns out to have only x-components (4.52).

We can also find the momentum density, i.e., field momentum per unit volume that the field possesses at the object position, $(c^2, 0, 0)$ in the accelerated frame S, at object proper time τ and estimated from the view point of I_*. For this purpose, we divide

$\mathbf{S}_*^{zp}(\tau)$ by c^2, and obtain

$$\mathbf{g}_*^{zp}(\tau) = \frac{\mathbf{S}_*^{zp}(\tau)}{c^2} = -\hat{x}\frac{1}{4\pi c}\frac{8\pi}{3}\sinh\left(\frac{2a\tau}{c}\right)\int \frac{\hbar\omega^3}{2\pi^2 c^3}d\omega. \tag{4.53}$$

The total amount of momentum due to the ZPF radiation inside the volume of the object and evaluated in the laboratory I_* frame is simply $\mathbf{g}_*^{zp}$ multiplied by the volume V_*, which gives

$$\mathbf{p}_*^{zp}(\tau) = V_*\mathbf{g}_*^{zp} = \frac{V_0}{\gamma_\tau}\mathbf{g}_*^{zp}(\tau) = -\hat{x}\frac{4V_0}{3c}\beta_\tau\gamma_\tau \int \frac{\hbar\omega^3}{2\pi^2 c^3}d\omega \tag{4.54}$$

where equations (4.7) and (4.8) and the relation $\sinh 2x = 2\sinh x\cosh x$ were used.

At proper time $\tau = 0$, the time in the laboratory inertial frame $t_* = \gamma_\tau\tau$ is of course also zero, and the Rindler frame S and the laboratory frame I_* exactly coincide momentarily, and the object location, $(c^2/a, 0, 0)$ of S, matches the observer's position in his laboratory frame I_* as well. If the object is moving at a constant speed, the I_*-observer will find the ZPF momentum of the object to be time *independent* constant of motion. However, under the hyperbolic motion that we consider in this research, the object appears from the view point of the I_*-observer to be carrying a time *dependent* ZPF momentum $\mathbf{p}_*^{zp}$(4.54), due to the acceleration of the object. Since this ZPF momentum of the object as observed in the I_* frame varies with time, we can evaluate the time rate of change of this momentum,

$$\mathbf{f}_*^{zp} = \frac{d\mathbf{p}_*^{zp}}{dt_*} = \frac{1}{\gamma_\tau}\left.\frac{d\mathbf{p}_*^{zp}}{d\tau}\right|_{\tau=0} = -\hat{x}\left[\frac{4V_0}{3c^2}\int \frac{\hbar\omega^3}{2\pi^2 c^3}d\omega\right]\mathbf{a} \tag{4.55}$$

This $\mathbf{f}_*^{zp}$ is the force exerted on the object by the ZPF radiation as seen in the laboratory inertial frame I_* at $t_* = 0$. We note here that this force is directed in the opposite direction to the object's motion, and its magnitude is proportional to the object's acceleration.

Moreover, the scalar quantity,

$$m_i = \left[\frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \right] \tag{4.56}$$

has a dimension of mass and we interpret this as an expression of inertial mass arising from the interaction of the object with the ZPF. The numerical factor of $4/3$ has been neglected here since a covariant analysis in Ch.5 shows that this factor vanishes. We have also included in the expression above a frequency-dependent interaction function $\eta(\omega)$, [6] such that $0 \leq \eta(\omega) \leq 1$, indicating that only a fraction of the zero-point energy contained inside the object's proper volume V_0 might be interacting to contribute to the inertial mass m_i.

We evaluated the Poynting vector of the ZPF radiation field that an object under a constant acceleration (hyperbolic motion) sweeps through as seen from the laboratory frame I_*. From this Poynting vector, the force that ZPF background radiation fields exerts upon the accelerating object is determined. This force $\mathbf{f}_*^{zp}$ turns out to be in the opposite direction to the object's motion, and its magnitude is found proportional to the acceleration $\mathbf{a}$. This linear relation between the ZPF reactive force $\mathbf{f}_*^{zp}$ and acceleration $\mathbf{a}$ of the object is analogous to that between the temperature and acceleration in the Davies-Unruh effect, [7] implying that the ZPF possess a structure which reacts against acceleration. We conclude, based upon the above results, that this reactive force between the accelerated object and the ZPF background radiation is the origin of inertia.

[6] A recent development on this "efficiency" or "interaction" function $\eta(\omega)$ is given in the reference [31], where Rueda and Haisch tries to explain $\eta(\omega)$ in terms of the summations of all the resonant cavity modes broadened by the Lorentzian broadening factors.

[7] In both the Davies-Unruh effect and in our analysis, the results obtained are proportional to the acceleration of the object under hyperbolic motion. This result seems to stem from the property of the quantum vacuum that reacts against the acceleration. In this respect, our result and that of the Davies-Unruh effect appears to share the same roots. A derivation of Davies-Unruh effect is given in Appendix F using the same approach taken in the present research, and the calculations indeed look similar to those in Appendices C and D.

4.6 Momentum Content Approach

In this section, the ZPF *momentum content* within the object will be evaluated, rather than the *momentum flux* obtained in the previous section. Let $\mathbf{g}_*$ denote the momentum density inside the object under the hyperbolic motion. In a short time interval Δt_*, the momentum per unit volume in the interior of the object will increase by an amount $\Delta\mathbf{g}_*$ due to its accelerated motion.This increase in the object's momentum content must come from the surrounding ZPF, that is, the amount of the background ZPF swept through by the object in the same time interval Δt_*. This amount of the momentum flux $-\mathbf{g}_*^{zp}$ is the quantity we have just calculated in the previous section, and it is expected that the relation

$$\mathbf{g}_* = -\mathbf{g}_*^{zp} \tag{4.57}$$

is to be obtained. For this purpose, we like to evaluate the following quantity,

$$\mathbf{g}_* = \hat{x} g_{*x} = \frac{\mathbf{S}_*}{c^2} = \hat{x}\frac{1}{c^2}\frac{c}{4\pi}\left\langle 0 \left| \mathbf{E}_*^{zp}(0,\tau) \times \mathbf{B}_*^{zp}(0,\tau) \right| 0 \right\rangle_x \tag{4.58}$$

where we only consider the x-component of the Poynting vector since the motion is in this direction. The momentum density $\mathbf{g}_*$ and the associated Poynting vector $\mathbf{S}_*$ are to be evaluated in the laboratory frame I_* at the object's position at its proper time τ, i.e.,

$$t_* = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right), \quad x_* = \mathbf{R}_*(\tau)\cdot\hat{x} = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right), \quad y_* = 0, \quad z_* = 0, \tag{4.59}$$

as before, since the lab frame I_* is the ultimate reference frame where all the physical quantities are to be evaluated.

However, since we like to evaluate the ZPF momentum content inside the body of the object which is instantaneously at rest in the I_τ frame, the integrals are to be taken in the object's instantaneous rest frame I_τ. Therefore, in order to express the quantity $\left\langle 0 \left| \mathbf{E}_*^{zp}(0,\tau) \times \mathbf{B}_*^{zp}(0,\tau) \right| 0 \right\rangle_x$ in terms of the ZPF components in the I_τ frame, we apply (inverse) Lorentz transformations to the ZPF in the I_τ frame, namely

$$\begin{aligned} E_1 &= E_1' & B_1 &= B_1' \\ E_2 &= \gamma(E_2' + \beta B_3') & B_2 &= \gamma(B_2' - \beta E_3') \\ E_3 &= \gamma(E_3' - \beta B_2') & B_3 &= \gamma(B_3' + \beta E_2') \end{aligned}$$

and obtain for the fields

$$\mathbf{E}_*^{zp}(0,\tau) = \sum_{\lambda=1}^{2} \int d^3k \left\{ \hat{x}\hat{\epsilon}_x + \hat{y}\gamma_\tau[\hat{\epsilon}_y + \beta_\tau(\hat{k}\times\hat{\epsilon})_z] + \hat{z}\gamma_\tau[\hat{\epsilon}_z - \beta_\tau(\hat{k}\times\hat{\epsilon})_y] \right\} H_{zp}(\omega)$$
$$\times \left\{ \alpha(\mathbf{k},\lambda) \exp[i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)] + \alpha^\dagger(\mathbf{k},\lambda) \exp[-i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)] \right\}, \qquad (4.60)$$

$$\mathbf{B}_*^{zp}(0,\tau) = \sum_{\lambda=1}^{2} \int d^3k \left\{ \hat{x}(\hat{k}\times\hat{\epsilon})_x + \hat{y}\gamma_\tau[(\hat{k}\times\hat{\epsilon})_y - \beta_\tau\hat{\epsilon}_z] + \hat{z}\gamma_\tau[(\hat{k}\times\hat{\epsilon})_z + \beta_\tau\hat{\epsilon}_y] \right\}$$
$$\times H_{zp}(\omega) \left\{ \alpha(\mathbf{k},\lambda) \exp[i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)] + \alpha^\dagger(\mathbf{k},\lambda) \exp[-i(\mathbf{k}\cdot\mathbf{R}_* - \omega t_*)] \right\}. \qquad (4.61)$$

With this Lorentz transformed ZPF components, we are going to evaluate the ZPF expectation values,

$$\left\langle 0 \left| \mathbf{E}_*^{zp}(0,\tau) \times \mathbf{B}_*^{zp}(0,\tau) \right| 0 \right\rangle_x = \left\langle 0 \left| E_{y*}B_{z*} - E_{z*}B_{y*} \right| 0 \right\rangle \qquad (4.62)$$

that we use in the evaluation of the ZPF momentum density $\mathbf{g}_*$ and the ZPF momentum $\mathbf{p}_*$. The mathematical treatment of this momentum flux approach is similar to that of the momentum flux approach, but these two methods are independent of each other.

Detailed calculations of the expectation values for all nine ZPF components $\left\langle 0 \left| E_{i*}B_{j*} \right| 0 \right\rangle$ where $i, j = x, y, z$ are given in Appendix D, and it turns out, as expected, that all the terms vanish except $\left\langle 0 \left| E_{y*}B_{z*} \right| 0 \right\rangle$ and $\left\langle 0 \left| E_{z*}B_{y*} \right| 0 \right\rangle$, and that these two terms are related by

$$\left\langle 0 \left| E_{y*}B_{z*} \right| 0 \right\rangle = -\left\langle 0 \left| E_{z*}B_{y*} \right| 0 \right\rangle, \qquad (4.63)$$

a similar result as that obtained in the momentum-flux approach. The non-vanishing expectation value is found to be

$$\left\langle 0 \left| E_{y*} B_{z*} \right| 0 \right\rangle = \frac{8\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{4\pi^2 c^3} d\omega, \tag{4.64}$$

which yields

$$\begin{aligned} \left\langle 0 \left| \mathbf{E}_*^{zp}(0,\tau) \times \mathbf{B}_*^{zp}(0,\tau) \right| 0 \right\rangle_x &= \left\langle 0 \left| E_{y*} B_{z*} - E_{z*} B_{y*} \right| 0 \right\rangle \\ &= 2 \left\langle 0 \left| E_{y*} B_{z*} \right| 0 \right\rangle \\ &= \frac{16\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{4\pi^2 c^3} d\omega, \end{aligned} \tag{4.65}$$

and the corresponding ZPF Poynting vector

$$\begin{aligned} \mathbf{S}_* &= \hat{x} \frac{c}{4\pi} \left\langle 0 \left| E_*^{zp}(0,\tau) \times B_*^{zp}(0,\tau) \right| 0 \right\rangle_x \\ &= \hat{x} \frac{4c}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{4\pi^2 c^3} d\omega, \end{aligned} \tag{4.66}$$

which represents the ZPF energy contained inside the uniformly accelerating object's body per unit area per unit time as seen from the inertial observer at rest in the laboratory frame I_*.

Following the same procedures as the momentum-flux case, we can find the ZPF momentum density of the object,

$$\mathbf{g}_* = \frac{\mathbf{S}_*}{c^2} = \hat{x} \frac{4}{3c} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{4\pi^2 c^3} d\omega, \tag{4.67}$$

and the ZPF momentum contained momentarily inside the volume of the object as seen from the inertial observer in I_* frame, which can be found by multiplying the ZPF momentum density above by the object's volume in I_*, $V_* = V_0/\gamma$,

$$
\begin{aligned}
\mathbf{p}_* &= \mathbf{g}_* V_* \\
&= \hat{x}\frac{V_0}{\gamma}\frac{4}{3c}\sinh\left(\frac{2a\tau}{c}\right)\int\frac{\hbar\omega^3}{4\pi^2c^3}d\omega \\
&= \hat{x}\frac{4V_0}{3c^2}c\beta\gamma\int\frac{\hbar\omega^3}{2\pi^2c^3}d\omega,
\end{aligned}
\tag{4.68}
$$

where the relation $\sinh 2x = 2\sinh x\cosh x$ was used again, together with the relation $\cosh(a\tau/c) = \gamma$ and $\sinh(a\tau/c) = \beta\gamma$. The rate of change of this momentum with respect to time is the force $\mathbf{f}_*$ the object under hyperbolic motion is exerting against the ZPF,

$$
\mathbf{f}_* = \frac{d\mathbf{p}_*}{dt_*} = \frac{1}{\gamma_\tau}\frac{d\mathbf{p}_*}{d\tau}\bigg|_{\tau=0} = \hat{x}\left[\frac{4V_0}{3c^2}\int\frac{\hbar\omega^3}{2\pi^2c^3}d\omega\right]\mathbf{a}. \tag{4.69}
$$

Comparing this expression with the $\mathbf{f}_*^{zp}$ obtained in the previous section, Eq.(4.55), we immediately notice that $\mathbf{f}_*^{zp} = -\mathbf{f}_*$, which is a reinstatement of Newton's third law, the ZPF applies equal and opposite force against the accelerating object.

5 Covariant Approach

In the previous sections, the electromagnetic zero-point-field (ZPF) Poynting vector $\mathbf{S}^{zp} = \frac{c}{4\pi}(\mathbf{E}^{zp} \times \mathbf{B}^{zp})$ and its vacuum expectation values $\frac{c}{4\pi} < 0|E_i^{zp} B_j^{zp}|0 >$ were evaluated. In this section, these quantities are to be evaluated using a covariant method. It will be shown that the factor of $4/3$ for an expression of inertial mass, obtained earlier in the non-covariant method, vanishes in this fully covariant approach. This is expected because the relativistic momentum of an object with mass m should be $\gamma m v$, not $4/3\gamma m v$.

Historically, it was Lorentz[32] and Abraham[33] who obtained this $4/3$ factor in their study of the classical theory of an electron, which gave incorrect kinematical relationship between the momentum and velocity of an electron. However, it was shown later by Fermi[34][35], Wilson[36], Kwal[37], and Rohrlich[38] that the extra factor of $4/3$ should not be there for the momentum of an electron. This incorrect factor comes from the incorrect definitions of relativistic energy and momentum. In our analysis, it will be shown, following the approach by Rohrlich[39], and Rueda and Haisch[7], that this factor of $4/3$ indeed does vanish.

5.1 Covariant Approach for the Evaluation of the Poynting Vector

In this section, the Poynting vector is evaluated in a covariant method. The Poynting vector $\mathbf{S}$ is an element of the symmetrical *electromagnetic energy-momentum tensor*

$$\Theta^{\mu\nu} = \begin{pmatrix} -U & -S_x/c & -S_y/c & -S_z/c \\ -S_x/c & T_{xx} & T_{xy} & T_{xz} \\ -S_y/c & T_{yx} & T_{yy} & T_{yz} \\ -S_z/c & T_{zx} & T_{zy} & T_{zz} \end{pmatrix} \tag{5.1}$$

In the above, the time and mixed space-time components are

$$\Theta^{00} = \frac{1}{8\pi}\left(E^2 + B^2\right) \equiv -U, \tag{5.2}$$

and

$$\Theta^{0i} = -\frac{1}{4\pi}\left(\mathbf{E} \times \mathbf{B}\right)_i, \tag{5.3}$$

where U is the electromagnetic energy density and

$$\mathbf{S} \equiv \frac{c}{4\pi}\left(\mathbf{E} \times \mathbf{B}\right) \tag{5.4}$$

is the *Poynting vector*, which is also an energy flux density.

The space part of the tensor Θ^{ij} is called the *Maxwell stress tensor* whose components are given as

$$T_{ij} = \frac{1}{4\pi}\left[E_i E_j + B_i B_j - \frac{1}{2}(E^2 + B^2)\delta_{ij}\right]. \tag{5.5}$$

Now let us consider the quantity,

$$P^\mu \equiv \frac{1}{c}\int \Theta^{\mu\nu} d\sigma_\nu, \tag{5.6}$$

the integration of the electromagnetic energy tensor over a spacelike plane σ given by the equation

$$n^\mu x_\mu + c\tau = 0, \tag{5.7}$$

and n^μ is the unit normal vector of the plane, which is necessarily timelike,

$$n_\mu n^\mu = -1. \tag{5.8}$$

Any instant of an inertial observer is characterized by this spacelike plane σ and the unit normal n^ν. For example, when $n^\nu = (1;0,0,0)$, $\tau = t$, then the spacelike plane σ describes xyz-plane at the instant t. If $n^\mu = v^\mu/c$, where v^μ is the four-velocity with which the inertial frame K' is moving with respect to K, the plane σ is tilted in K, and a Lorentz transformation to K' transforms σ to the plane $\tau = t'$ in K', which coincides with the xyz-plane in K'. Thus, the choice of $n^\mu = v^\mu/c$ describes the three-space $t' = \tau$ in K', as seen by K.

The surface element on such a plane is given by the vector

$$d\sigma^\mu = n^\mu d\sigma, \tag{5.9}$$

and its invariant area element can most easily be determined by the use of the unit normal $n^\mu = (1;0,0,0)$ in the example above as,

$$d\sigma = -n_\mu d\sigma^\mu = dxdydz, \tag{5.10}$$

because in the rest frame where the unit normal is necessarily $n^\mu = (1;0,0,0)$, the spacelike surface σ is a simple *plane* perpendicular to the time axis, i.e., the xyz-plane whose volume element is dxdydz.

Now let us go back to the expression (5.6). In the particular Lorentz frame whose surface normal is given by $n^\nu = (1;0,0,0)$, the components of P^μ can be given explicitly as

$$P^{\mu(0)} = \left(\frac{1}{c}W^{(0)}, \mathbf{P}^{(0)}\right) \tag{5.11}$$

with

$$W^{(0)} = \int U^{(0)} d^3x \tag{5.12}$$

and

$$\mathbf{P}^{(0)} = \frac{1}{c^2}\int \mathbf{S}^{(0)} d^3x. \tag{5.13}$$

In the case of interest to us, that is, the velocity is along the positive x-direction, the surface normal is given by

$$n^{\nu} = (\gamma; \gamma\beta\hat{\mathbf{n}}), \tag{5.14}$$

where $\hat{\mathbf{n}}$ is a unit three vector, and the equation (5.6) takes the following forms:

$$P^{\mu} = \left(\frac{1}{c}W, \mathbf{P}\right) \tag{5.15}$$

with

$$W = \gamma \int U d\sigma - \frac{\gamma\beta}{c} \int \mathbf{S} \cdot \hat{\mathbf{n}} d\sigma \tag{5.16}$$

and

$$\mathbf{P} = \frac{\gamma}{c^2} \int \mathbf{S} d\sigma + \frac{\gamma\beta}{c} \int \overleftrightarrow{\mathbf{T}} \cdot \hat{\mathbf{n}} d\sigma, \tag{5.17}$$

where $\overleftrightarrow{\mathbf{T}}$ is a Maxwell stress tensor whose components are given by Eq. (5.5).

The derivation of the above equations is given in Appendix E. At this point, we identify P^{μ} of equation (5.6) as the momentum four-vector of the electromagnetic field. Note in passing that extra terms appear in (5.16) and (5.17), which can also be obtained from the corresponding Lorentz transformation with a velocity $\mathbf{v}$ whose magnitude is $\gamma\beta$ and whose direction is $\hat{\mathbf{n}}$. Also, it is to be noticed that the two expressions coincide if and only if γ is 1.

Abraham and Lorentz used the expressions (5.12) and (5.13) as their definitions for the energy density and the momentum, and they were led to the incorrect result for the momentum of an electron which includes the incorrect factor of $4/3$. However, as we have already shown, the equations (5.12) and (5.13) are only valid in the particular Lorentz frame where γ is 1. It will be shown that with the use of the correct forms (5.16) and (5.17) for the energy density and the momentum, this incorrect factor of $4/3$ is reduced to unity.

5.2 Evaluation of the ZPF Momentum Content

We are now ready to evaluate the momentum in a covariant method. This can be done in two different approaches, i.e., the momentum-flux approach and the momentum-content approach, just in the same manner as in the non-covariant method. However, since the two treatments are very similar except the signs, only the momentum-content approach will be shown here. The expressions that we need to evaluate are

$$P^0 = \frac{\gamma}{c} \int (U - \mathbf{v} \cdot \mathbf{g})\, d^3\sigma \tag{5.18}$$

$$\mathbf{p} = \gamma \int \left(\mathbf{g} + \frac{\overleftrightarrow{\mathbf{T}} \cdot \mathbf{v}}{c^2} \right) d^3\sigma \tag{5.19}$$

In the above expressions, the integration is taken over the volume of the object with the volume element $d^3\sigma$, which is an invariant hypersurface element equal to a 3-space volume element. Since we assume that the volume of the object is so small, the integrand is considered constant and we just multiply it by the object volume V_0, as has been done in the non-covariant method. Therefore, we evaluate the quantity

$$\mathbf{p}_* = \gamma \left(\mathbf{g}_* + \frac{\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}_*}{c^2} \right) V_0 \tag{5.20}$$

This is the momentum inside the uniformly accelerating object in the comoving frame I_τ as seen from the lab inertial frame I_*. $\overleftrightarrow{\mathbf{T}}$ in the above equation is the Maxwell stress tensor whose components are given by the Eq.(5.5). Therefore, the dot product of $\overleftrightarrow{\mathbf{T}}_*$ with the velocity $\mathbf{v} = v\hat{x}$ in the above equation yields the column vector

$$\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v} = \begin{bmatrix} T_{xx*}v \\ T_{yx*}v \\ T_{zx*}v \end{bmatrix} = (\hat{x}T_{xx*} + \hat{y}T_{yx*} + \hat{z}T_{zx*})v \tag{5.21}$$

with T_{ij*} given by (5.5). The components of the ZPF included in the expression for T_{ij*} are the fields inside the object as seen from the lab frame I_*. Therefore, to obtain these zero-point field components, we apply the Lorentz transformation from the object in the instantaneous comoving frame I_τ to the inertial lab frame I_*. This gives, for the zero-point field components,

$$\mathbf{E}_*^{zp}(\tau; c^2/a, 0, 0) = \sum_{\lambda=1}^{2} \int d^3k \left\{ \hat{x}\hat{\epsilon}_x + \hat{y}\gamma_\tau[\hat{\epsilon}_y + \beta_\tau(\hat{k}\times\hat{\epsilon})_z] + \hat{z}\gamma_\tau[\hat{\epsilon}_z - \beta_\tau(\hat{k}\times\hat{\epsilon})_y] \right\}$$
$$\times H_{zp}(\omega) \left\{ \alpha(\mathbf{k},\lambda)\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{R}_*) + \alpha^\dagger(\mathbf{k},\lambda)\exp(i\omega t - i\mathbf{k}\cdot\mathbf{R}_*) \right\}, \tag{5.22}$$

$$\mathbf{B}_*^{zp}(\tau; c^2/a, 0, 0) = \sum_{\lambda=1}^{2} \int d^3k \left\{ \hat{x}(\hat{k}\times\hat{\epsilon})_x + \hat{y}\gamma_\tau[(\hat{k}\times\hat{\epsilon})_y - \beta_\tau\hat{\epsilon}_z] + \hat{z}\gamma_\tau[(\hat{k}\times\hat{\epsilon})_z + \beta_\tau\hat{\epsilon}_y] \right\}$$
$$\times H_{zp}(\omega) \left\{ \alpha(\mathbf{k},\lambda)\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{R}_*) + \alpha^\dagger(\mathbf{k},\lambda)\exp(i\omega t - i\mathbf{k}\cdot\mathbf{R}_*) \right\}. \tag{5.23}$$

These are the zero-point field components that are contained inside the object's proper volume in the comoving frame I_τ as seen from the observer in the inertial laboratory frame I_*. With these ZPF components, we are now ready to evaluate the vacuum expectation value for each term in the Eq.(5.21). It is shown first that the y and z components of the expectation values for $\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}$ vanish. This is physically reasonable since the object is moving in the positive x-direction.

We have, for the y-component,

$$\left\langle 0 \left| T_{yx*} \right| 0 \right\rangle = \frac{1}{4\pi} \left\langle 0 \left| E_{y*}E_{x*} + B_{y*}B_{x*} \right| 0 \right\rangle \tag{5.24}$$

and the first term gives

$$\left\langle 0 \left| E_{y*}E_{x*} \right| 0 \right\rangle = \left\langle 0 \left| \gamma_\tau[E_{y\tau} + \beta B_{z\tau}]E_{x\tau} \right| 0 \right\rangle$$
$$= \gamma_\tau \left\langle 0 \left| E_{y\tau}E_{x\tau} \right| 0 \right\rangle + \gamma_\tau\beta_\tau \left\langle 0 \left| B_{z\tau}E_{x\tau} \right| 0 \right\rangle. \tag{5.25}$$

The expectation value $\left\langle 0 \left| E_{y\tau} E_{x\tau} \right| 0 \right\rangle$, however, involves the factor

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y \hat{\epsilon}_x = -\hat{k}_y \hat{k}_x = -\hat{k}_x \hat{k}_y \tag{5.26}$$

which was shown previously to vanish after angular integration. Similarly, the second expectation value $\langle 0 | B_{z\tau} E_{x\tau} | 0 \rangle$ involves the factor

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x \left(\hat{k} \times \hat{\epsilon} \right)_z = -\hat{k}_y \tag{5.27}$$

which also vanishes upon integration. The second term of the y-component of $\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}$ in the Eq.(5.24) can also be shown to vanish in a similar manner. We have

$$\begin{aligned} \left\langle 0 \left| B_{y*} B_{x*} \right| 0 \right\rangle &= \left\langle 0 \left| \gamma_\tau [B_{y\tau} - \beta E_{z\tau}] B_{x\tau} \right| 0 \right\rangle \\ &= \gamma_\tau \left\langle 0 \left| B_{y\tau} B_{x\tau} \right| 0 \right\rangle - \gamma_\tau \beta_\tau \langle 0 | E_{z\tau} B_{x\tau} | 0 \rangle . \end{aligned} \tag{5.28}$$

The first term $\left\langle 0 \left| B_{y\tau} B_{x\tau} \right| 0 \right\rangle$ involves the factor

$$\sum_{\lambda=1}^{2} \left(\hat{k} \times \hat{\epsilon} \right)_y \left(\hat{k} \times \hat{\epsilon} \right)_x = -\hat{k}_y \hat{k}_x, \tag{5.29}$$

and the second term $\langle 0 | E_{z\tau} B_{x\tau} | 0 \rangle$ includes

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z \left(\hat{k} \times \hat{\epsilon} \right)_x = \hat{k}_y, \tag{5.30}$$

both of which have already been shown to vanish. Therefore, as we have expected, the y-components of $\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}$, i.e., the equation (5.24) vanish. It can also be shown easily, in a very similar manner, that the z-components of $\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}$ also vanish, which leads us to the conclusion that the only contribution to $\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}$ comes from the x-component. We have for this x-component

$$\begin{aligned}\langle 0\,|T_{xx*}|\,0\rangle &= \frac{1}{4\pi}\left\langle 0\left|E_{x*}E_{x*}+B_{x*}B_{x*}-\frac{1}{2}\left(E_*^2+B_*^2\right)\right|0\right\rangle \\ &= \frac{1}{4\pi}\left\langle 0\left|E_{x*}^2+B_{x*}^2\right|0\right\rangle - \frac{1}{8\pi}\left\langle 0\left|E_*^2+B_*^2\right|0\right\rangle \qquad (5.31)\end{aligned}$$

where

$$E_*^2 = E_{x*}^2 + E_{y*}^2 + E_{z*}^2, \qquad (5.32)$$

and

$$B_*^2 = B_{x*}^2 + B_{y*}^2 + B_{z*}^2 \qquad (5.33)$$

Each component of the zero-point fields is given by the Lorentz transformations (5.22) and (5.23) as

$$\left\langle 0\left|E_{x*}^2\right|0\right\rangle = \left\langle 0\left|E_{x\tau}^2\right|0\right\rangle \qquad (5.34)$$

$$\left\langle 0\left|B_{x*}^2\right|0\right\rangle = \left\langle 0\left|B_{x\tau}^2\right|0\right\rangle \qquad (5.35)$$

$$\begin{aligned}\left\langle 0\left|E_{y*}^2\right|0\right\rangle &= \left\langle 0\left|\gamma_\tau(E_{y\tau}+\beta_\tau B_{z\tau})\gamma_\tau(E_{y\tau}+\beta_\tau B_{z\tau})\right|0\right\rangle \\ &= \gamma_\tau^2\left\langle 0\left|E_{y\tau}^2\right|0\right\rangle + \gamma_\tau^2\beta_\tau^2\left\langle 0\left|B_{z\tau}^2\right|0\right\rangle + 2\gamma_\tau^2\beta_\tau\left\langle 0\left|E_{y\tau}B_{z\tau}\right|0\right\rangle \qquad (5.36)\end{aligned}$$

and similarly for other components

$$\left\langle 0\left|B_{y*}^2\right|0\right\rangle = \gamma_\tau^2\left\langle 0\left|B_{y\tau}^2\right|0\right\rangle + \gamma_\tau^2\beta_\tau^2\left\langle 0\left|E_{z\tau}^2\right|0\right\rangle - 2\gamma_\tau^2\beta_\tau\left\langle 0\left|E_{z\tau}B_{y\tau}\right|0\right\rangle \qquad (5.37)$$

$$\left\langle 0\left|E_{z*}^2\right|0\right\rangle = \gamma_\tau^2\left\langle 0\left|E_{z\tau}^2\right|0\right\rangle + \gamma_\tau^2\beta_\tau^2\left\langle 0\left|B_{y\tau}^2\right|0\right\rangle - 2\gamma_\tau^2\beta_\tau\left\langle 0\left|E_{z\tau}B_{y\tau}\right|0\right\rangle \qquad (5.38)$$

$$\left\langle 0\left|B_{z*}^2\right|0\right\rangle = \gamma_\tau^2\left\langle 0\left|B_{z\tau}^2\right|0\right\rangle + \gamma_\tau^2\beta_\tau^2\left\langle 0\left|E_{y\tau}^2\right|0\right\rangle - 2\gamma_\tau^2\beta_\tau\left\langle 0\left|E_{y\tau}B_{z\tau}\right|0\right\rangle \qquad (5.39)$$

Using these relationships, we can now evaluate the terms in the Eq.(5.31). From (5.34) and (5.35), we have for the first term in (5.31),

$$\frac{1}{4\pi}\left\langle 0\left|E_{x*}^2+B_{x*}^2\right|0\right\rangle = \frac{1}{4\pi}\left\langle 0\left|E_{x\tau}^2+B_{x\tau}^2\right|0\right\rangle$$
$$= \frac{1}{12\pi}\left\langle 0\left|E_{\tau}^2+B_{\tau}^2\right|0\right\rangle, \tag{5.40}$$

where the relation

$$\left\langle 0\left|E_{i\tau}^2\right|0\right\rangle = \frac{1}{3}\left\langle 0\left|E_{\tau}^2\right|0\right\rangle = \frac{1}{3}\left\langle 0\left|B_{\tau}^2\right|0\right\rangle = \left\langle 0\left|B_{i\tau}^2\right|0\right\rangle \tag{5.41}$$

with $i = x, y, z$, was used in the last step. Since we also have

$$U = \frac{1}{8\pi}\left\langle 0\left|E_{\tau}^2+B_{\tau}^2\right|0\right\rangle = \int \frac{\hbar\omega^3}{2\pi^2c^3}d\omega, \tag{5.42}$$

we obtain

$$\left\langle 0\left|E_{\tau}^2+B_{\tau}^2\right|0\right\rangle = 8\pi\int \frac{\hbar\omega^3}{2\pi^2c^3}d\omega \tag{5.43}$$

Upon substituting the above equation into (5.40), we find that the first term of (5.31) becomes

$$\frac{1}{4\pi}\left\langle 0\left|E_{x*}^2+B_{x*}^2\right|0\right\rangle = \frac{2}{3}\int \frac{\hbar\omega^3}{2\pi^2c^3}d\omega. \tag{5.44}$$

For the evaluation of the second term of (5.31),

$$\frac{1}{8\pi}\left\langle 0\left|E_{*}^2+B_{*}^2\right|0\right\rangle = \frac{1}{8\pi}\left\langle 0\left|E_{x*}^2+B_{x*}^2+E_{y*}^2+B_{y*}^2+E_{z*}^2+B_{z*}^2\right|0\right\rangle, \tag{5.45}$$

we substitute the relations (5.34) to (5.39) into the above, which gives

$$\frac{1}{8\pi}\left\langle 0\left|E_{*}^2+B_{*}^2\right|0\right\rangle$$
$$= \frac{1}{8\pi}\Big[\left\langle 0\left|E_{x\tau}^2\right|0\right\rangle + \left\langle 0\left|B_{x\tau}^2\right|0\right\rangle + \gamma_\tau^2\left\langle 0\left|E_{y\tau}^2+E_{z\tau}^2+B_{y\tau}^2+B_{z\tau}^2\right|0\right\rangle$$
$$+\gamma_\tau^2\beta_\tau^2\left\langle 0\left|E_{y\tau}^2+E_{z\tau}^2+B_{y\tau}^2+B_{z\tau}^2\right|0\right\rangle + 2\gamma_\tau^2\beta_\tau\left(2\left\langle 0\left|E_{y\tau}B_{z\tau}-E_{z\tau}B_{y\tau}\right|0\right\rangle\right)\Big] \tag{5.46}$$

Both terms in the last equality are zero since $\left\langle 0 \left| E_{y\tau} B_{z\tau} \right| 0 \right\rangle$ involves the factor

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y \left(\hat{k} \times \hat{\epsilon}\right)_z = \hat{k}_x, \tag{5.47}$$

and the second term $\left\langle 0 \left| E_{z\tau} B_{y\tau} \right| 0 \right\rangle$ involves the factor

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z \left(\hat{k} \times \hat{\epsilon}\right)_y = -\hat{k}_x, \tag{5.48}$$

both of which have been shown previously to vanish after angular integration. This result is actually an expected one, since the term in triangular brackets in the last equality is proportional to the x-component of the zero-point field Poynting vector in I_τ and this frame is comoving with the object. For the evaluation of other terms in (5.46), we combine the relations (5.41) through (5.43) and obtain

$$\left\langle 0 \left| E_{i\tau}^2 \right| 0 \right\rangle = \left\langle 0 \left| B_{i\tau}^2 \right| 0 \right\rangle = \frac{4\pi}{3} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \tag{5.49}$$

Since the above relation holds for any components for $i = x, y, z$, the expectation values of all the squared fields have the same value given above, and the equation (5.46) simplifies to

$$\begin{aligned} \frac{1}{8\pi} \left\langle 0 \left| E_*^2 + B_*^2 \right| 0 \right\rangle &= \frac{1}{8\pi} \frac{4\pi}{3} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \left[1 + 1 + 4\gamma_\tau^2 + 4\gamma_\tau^2 \beta_\tau^2\right] \\ &= \frac{1}{3} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \left(1 + 2\gamma_\tau^2 + 2\gamma_\tau^2 \beta_\tau^2\right) \end{aligned} \tag{5.50}$$

By combining the above results, i.e., Eq(5.44) and Eq(5.50), we can now evaluate

$\langle 0 | T_{xx*} | 0 \rangle$. The equation (5.31) now becomes

$$\begin{aligned}\langle 0 | T_{xx*} | 0 \rangle &= \frac{1}{4\pi} \left\langle 0 \left| E_{x*}^2 + B_{x*}^2 \right| 0 \right\rangle - \frac{1}{8\pi} \left\langle 0 \left| E_*^2 + B_*^2 \right| 0 \right\rangle \\ &= \frac{2}{3} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega - \frac{1}{3} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \left(1 + 2\gamma_\tau^2 + 2\gamma_\tau^2 \beta_\tau^2\right) \\ &= \frac{1}{3} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \left(1 - 2\gamma_\tau^2 - 2\gamma_\tau^2 \beta_\tau^2\right)\end{aligned} \tag{5.51}$$

where (5.44) and (5.50) were used. This gives

$$\hat{x} \frac{T_{xx} v}{c^2} = \hat{x} \frac{1}{3} c\beta_\tau \frac{1}{c^2} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \left(1 - 2\gamma_\tau^2 - 2\gamma_\tau^2 \beta_\tau^2\right) \tag{5.52}$$

which is used to find the momentum, together with the value of $\mathbf{g}_*$, previously found to be

$$\mathbf{g}_* = \hat{x} \frac{4}{3} c\beta_\tau \gamma_\tau^2 \frac{1}{c^2} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \tag{5.53}$$

Using the two results above, we can finally obtain for the momentum

$$\begin{aligned}\mathbf{p}_* &= \gamma \left(\mathbf{g}_* + \frac{\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}_*}{c^2} \right) V_0 \\ &= \hat{x} \gamma V_0 c\beta_\tau \frac{1}{c^2} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \frac{1}{3} \left[4\gamma_\tau^2 + 1 - 2\gamma_\tau^2 - 2\gamma_\tau^2 \beta_\tau^2\right] \\ &= \hat{x} \gamma V_0 c\beta_\tau \frac{1}{c^2} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \frac{1}{3} \left[1 + 2\gamma_\tau^2 \left(1 - \beta_\tau^2\right)\right] \\ &= \hat{x} \gamma V_0 c\beta_\tau \frac{1}{c^2} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega\end{aligned} \tag{5.54}$$

where the last equality is due to the cancellation $\gamma_\tau^2(1 - \beta_\tau^2) = 1$. We note here that the extra factor of $4/3$ we obtained earlier in a non-covariant method vanishes as expected in this covariant approach.

We can also easily check the zero-component of the momentum four-vector, given from Eq.(5.18) as

$$P^0 = \frac{\gamma_\tau}{c}\int (U - \mathbf{v}\cdot\mathbf{g}_*)\, d\sigma$$
$$= \frac{\gamma_\tau}{c}\left[\frac{\left\langle 0\left|E_*^2 + B_*^2\right|0\right\rangle}{8\pi} - c\beta_\tau g_*\right] V_0 \tag{5.55}$$

Since both terms in the above equation have already been found in (5.50) and (5.53), after substituting these results, we obtain

$$P^0 = \frac{\gamma_\tau V_0}{c}\int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \frac{1}{3}\left(1 + 2\gamma_\tau^2 + 2\gamma_\tau^2\beta_\tau^2 - 4\gamma_\tau^2\beta_\tau^2\right)$$
$$= \frac{\gamma_\tau V_0}{c}\int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \frac{1}{3}\left[1 + 2\gamma_\tau^2\left(1 - \beta_\tau^2\right)\right]$$
$$= \frac{\gamma_\tau V_0}{c}\int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \tag{5.56}$$

The inertia reaction force that is exerted upon the object by the ZPF as seen in I_* is

$$\mathbf{f}_*^{zp} = -\frac{d\mathbf{p}_*}{dt_*}$$
$$= -\frac{1}{\gamma_\tau}\frac{d\mathbf{p}_*}{dt_*}$$
$$= -\left(\frac{V_0}{c^2}\int \eta(\omega)\frac{\hbar\omega^3}{2\pi^2 c^3} d\omega\right)\mathbf{a} \tag{5.57}$$

With the identification of

$$m_i = \left[\frac{V_0}{c^2}\int \eta(\omega)\frac{\hbar\omega^3}{2\pi^2 c^3} d\omega\right] \tag{5.58}$$

as the inertial mass, we can obtain the standard four-momentum

$$P^\mu = m_i v^\mu = (m_i c\gamma_\tau; m_i \mathbf{v}\gamma_\tau) \tag{5.59}$$

where

$$v^{\mu} = (c\gamma_{\tau}; \mathbf{v}\gamma_{\tau}) \tag{5.60}$$

We can also obtain the expression for the four-force as

$$F^{\mu} = \frac{dP^{\mu}}{d\tau} = \frac{d}{d\tau}(m_i c\gamma_{\tau}; \mathbf{p}) = \gamma_{\tau}\left(\frac{1}{c}\frac{dE}{dt}; \frac{d\mathbf{p}}{dt}\right) = \gamma_{\tau}\left(\mathbf{f}\cdot\beta_{\tau}; \mathbf{f}\right) = (\mathbf{F}\cdot\beta_{\tau}; \mathbf{F}) \tag{5.61}$$

6 Summary of Contributions

This summary serves as a guide to show which part of this dissertation should be credited to the present author as its original contribution.

The basic idea of this dissertation, namely, inertia or inertial mass may have its origin in the interaction between ZPF and accelerating object was first proposed by Rueda, Haisch, and Puthoff [6], and later in a different approach by Rueda and Haisch [7], both of which within the framework of SED. This dissertation follows the same approach as [7], but all the calculations of the vacuum expectation values are done with the framework of QED, using the creation and annihilation operators. These calculation are shown in Ch.4 and in Appendix C and D, and these are the present author's original contributions.

In Ch.3, several differences between SED and QED formulations are explained. This has been done previously by Boyer[26], but the calculations and derivations are given here in more detail. Also, there exists a factor of $1/2$ discrepancy between SED and QED when the same calculation is performed in these two different methods. This point has been made explicit and explained in Appendix B.

In Ch.5, the ZPF reactive force and the inertial mass are derived in a covariant method. The basic techniques employed in this chapter comes from Rohrlich [39]. Actual calculations of the ZPF force has been done by Rueda and Haisch [7] in SED formulation. Calculations in QED format were performed for the first time by the present author.

Finally, derivations of Davies-Unruh effect is given in QED formulation in Appendix F. Boyer also did this [40] for some of the non-vanishing terms. In this dissertation, most terms have been calculated in more detail.

A Derivation of Polarization Formulae

A.1 Overview

The random radiation, as in (2.1) and (2.2), is expressed as a sum over two polarization states $\hat{\epsilon}(\mathbf{k}, \lambda)$. Let us consider the third unit vector $\hat{\epsilon}^3 = \hat{\mathbf{k}} = \mathbf{k}/k$, where $\mathbf{k}$ is the propagation vector. For each propagation vector, there correspond two mutually orthogonal polarization vectors $\hat{\epsilon}^1$ and $\hat{\epsilon}^2$. Then these three vectors form an orthonormal triad,

$$\hat{\epsilon}_i^\lambda \hat{\epsilon}_j^\lambda = \sum_{\lambda=1}^{3} \left(\hat{\epsilon}^\lambda\right)_i \left(\hat{\epsilon}^\lambda\right)_j = \sum_{\lambda=1}^{3} \hat{\epsilon}_i^\lambda \hat{\epsilon}_j^\lambda = \hat{\epsilon}_i^1 \hat{\epsilon}_j^1 + \hat{\epsilon}_i^2 \hat{\epsilon}_j^2 + \hat{\epsilon}_i^3 \hat{\epsilon}_j^3 = \delta_{ij}, \tag{A.1}$$

with the following properties

$$\hat{\epsilon}^l \cdot \hat{\epsilon}^m = \delta_{lm}, \quad l, m = 1, 2, 3 \tag{A.2}$$

$$\hat{\epsilon}^m \cdot \hat{k} = 0, \quad m = 1, 2 \tag{A.3}$$

$$\hat{k} = \hat{\epsilon}^1 \times \hat{\epsilon}^2. \tag{A.4}$$

In the above equations, the polarization components $\hat{\epsilon}_i^\lambda$ are to be understood as scalars. They are directional cosines, e.g., the projections of the polarization unit vectors onto the i-axis,

$$\hat{\epsilon}_i^\lambda = \hat{\epsilon}^\lambda \cdot \hat{x}_i, \quad \hat{x}_i = \hat{x}, \hat{y}, \hat{z} \tag{A.5}$$

This same convention will also be used with the $\hat{k}$ unit vector, i.e., $\hat{k}_x = \hat{k} \cdot \hat{x}$. We also omit from now on the superscripts λ for simplicity, and use the notation $\left(\hat{\epsilon}^\lambda\right)_i = \hat{\epsilon}_i$

In this appendix, the following three identities are derived:

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i \hat{\epsilon}_j = \delta_{ij} - \hat{k}_i \hat{k}_j \tag{A.6}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i \left(\hat{k} \times \hat{\epsilon}\right)_j = \sum_{k=x,y,z} \varepsilon_{ijk} \hat{k}_k \tag{A.7}$$

$$\sum_{\lambda=1}^{2} \left(\hat{k} \times \hat{\epsilon}\right)_i \left(\hat{k} \times \hat{\epsilon}\right)_j = \delta_{ij} - \hat{k}_i \hat{k}_j \tag{A.8}$$

The proof of each identity is given below.

A.2 Derivation of Each Formula

A.2.1 $\sum_{\lambda=1}^{2} \hat{\epsilon}_i \hat{\epsilon}_j = \delta_{ij} - \hat{k}_i \hat{k}_j$

$$proof: \quad \sum_{\lambda=1}^{2} \hat{\epsilon}_i \hat{\epsilon}_j = \left(\epsilon_i^1 \epsilon_j^1 + \epsilon_i^2 \epsilon_j^2 + \epsilon_i^3 \epsilon_j^3\right) - \epsilon_i^3 \epsilon_j^3 = \delta_{ij} - \hat{k}_i \hat{k}_j \tag{A.9}$$

A.2.2 $\sum_{\lambda=1}^{2} \hat{\epsilon}_i \left(\hat{k} \times \hat{\epsilon}\right)_j = \sum_{k=x,y,z} \varepsilon_{ijk} \hat{k}_k$

proof: Rewriting the cross product using the Levi-Civita symbol, we obtain

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i \left(\hat{k} \times \hat{\epsilon}\right)_j = \sum_{\lambda=1}^{2} \hat{\epsilon}_i \varepsilon_{jlm} \hat{k}_l \hat{\epsilon}_m \tag{A.10}$$

$$= \varepsilon_{jlm} \left(\hat{\epsilon}_i^1 \hat{k}_l \hat{\epsilon}_m^1 + \hat{\epsilon}_i^2 \hat{k}_l \hat{\epsilon}_m^2\right) \tag{A.11}$$

$$= \varepsilon_{jlm} \left(\hat{\epsilon}_i^1 \hat{\epsilon}_m^1 + \hat{\epsilon}_i^2 \hat{\epsilon}_m^2\right) \hat{k}_l \tag{A.12}$$

Using the previous identity (A.9), this equation becomes,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i \left(\hat{k} \times \hat{\epsilon}\right)_j = \varepsilon_{jlm} \left(\delta_{im} - \hat{k}_i \hat{k}_m\right) \hat{k}_l \tag{A.13}$$

$$= \delta_{im} \varepsilon_{jlm} \hat{k}_l - \varepsilon_{jlm} \hat{k}_i \hat{k}_l \hat{k}_m \tag{A.14}$$

The second term can be shown to reduce to zero, using the property of the Levi-Civita symbol as follows:

$$\varepsilon_{jlm}\hat{k}_i\hat{k}_l\hat{k}_m = \hat{k}_i\varepsilon_{jlm}\hat{k}_l\hat{k}_m = \hat{k}_i\left(\hat{k}\times\hat{k}\right)_j = 0 \tag{A.15}$$

Therefore, we have proven the identity

$$\sum_{\lambda=1}^{2}\hat{\epsilon}_i\left(\hat{k}\times\hat{\epsilon}\right)_j = \varepsilon_{jli}\hat{k}_l = \varepsilon_{ijl}\hat{k}_l = \sum_{k=x,y,z}\varepsilon_{ijk}\hat{k}_k \tag{A.16}$$

after relabeling the dummy index.

A.2.3 $\sum_{\lambda=1}^{2}\left(\hat{k}\times\hat{\epsilon}\right)_i\left(\hat{k}\times\hat{\epsilon}\right)_j = \delta_{ij} - \hat{k}_i\hat{k}_j$

proof: Developing the original equation, we get

$$\sum_{\lambda=1}^{2}\left(\hat{k}\times\hat{\epsilon}\right)_i\left(\hat{k}\times\hat{\epsilon}\right)_j = \left(\hat{k}\times\hat{\epsilon}^1\right)_i\left(\hat{k}\times\hat{\epsilon}^1\right)_j + \left(\hat{k}\times\hat{\epsilon}^2\right)_i\left(\hat{k}\times\hat{\epsilon}^2\right)_j \tag{A.17}$$

Using the cyclic identity (A.4), each of the $\left(\hat{k}\times\hat{\epsilon}\right)$ terms can be expressed in a single term as

$$\sum_{\lambda=1}^{2}\left(\hat{k}\times\hat{\epsilon}\right)_i\left(\hat{k}\times\hat{\epsilon}\right)_j = \hat{\epsilon}_i^2\hat{\epsilon}_j^2 + \left(-\hat{\epsilon}_i^1\right)\left(-\hat{\epsilon}_j^1\right) \tag{A.18}$$

$$= \hat{\epsilon}_i^1\hat{\epsilon}_j^1 + \hat{\epsilon}_i^2\hat{\epsilon}_j^2 \tag{A.19}$$

The last term in the above equation is just the sum of $\hat{\epsilon}_i\hat{\epsilon}_j$ over the polarization index λ, which is actually the first identity (A.9) that we proved in this section. Therefore, it is concluded that

$$\sum_{\lambda=1}^{2}\left(\hat{k}\times\hat{\epsilon}\right)_i\left(\hat{k}\times\hat{\epsilon}\right)_j = \sum_{\lambda=1}^{2}\hat{\epsilon}_i\hat{\epsilon}_j = \delta_{ij} - \hat{k}_i\hat{k}_j \tag{A.20}$$

B Derivation of the Spectral Function $H_{zp}(\omega)$

B.1 Overview

The electromagnetic zero-point radiation in its *classical* form is expressed in terms of a superposition of plane waves as [22],

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}(\mathbf{k}, \lambda) h_{zp}(\omega) \cos\left[\mathbf{k} \cdot \mathbf{r} - \omega t - \theta(\mathbf{k}, \lambda)\right], \tag{B.1}$$

$$\mathbf{B}(\mathbf{r}, t) = \sum_{\lambda=1}^{2} \int d^3k \left(\hat{k} \times \hat{\epsilon}\right) h_{zp}(\omega) \cos\left[\mathbf{k} \cdot \mathbf{r} - \omega t - \theta(\mathbf{k}, \lambda)\right], \tag{B.2}$$

and in the QED formulation as [26, 27]

$$\overline{\mathbf{E}}(\mathbf{r}, t) = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}(\mathbf{k}, \lambda) H_{zp}(\omega) \times \left[\alpha\left(\mathbf{k}, \lambda\right) \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r}) + \alpha^{\dagger}\left(\mathbf{k}, \lambda\right) \exp(i\omega t - i\mathbf{k} \cdot \mathbf{r})\right], \tag{B.3}$$

$$\overline{\mathbf{B}}(\mathbf{r}, t) = \sum_{\lambda=1}^{2} \int d^3k (\hat{k} \times \hat{\epsilon}) H_{zp}(\omega) \times \left[\alpha\left(\mathbf{k}, \lambda\right) \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{r}) + \alpha^{\dagger}\left(\mathbf{k}, \lambda\right) \exp(i\omega t - i\mathbf{k} \cdot \mathbf{r})\right]. \tag{B.4}$$

They are summed over two mutually perpendicular polarization states $\hat{\epsilon}\left(\mathbf{k}, \lambda\right)$. The two states are labeled by a dummy index $\lambda = 1, 2$, and orthogonal to the wave vector $\mathbf{k}$ as well (c.f., Appendix(A) for details.) In the classical case, the random phase $\theta(\mathbf{k}, \lambda)$, which is uniformly distributed over the interval $(0, 2\pi)$, independently of $\mathbf{k}$ and λ is introduced to generate the random nature of the radiation. In the QED case, the quantum annihilation and creation operators $\alpha\left(\mathbf{k}, \lambda\right)$ and $\alpha^{\dagger}\left(\mathbf{k}, \lambda\right)$ are used instead of the cosines.

Our main interest in this chapter, however, is on the spectral function $H_{zp}(\omega)$ and

$h_{zp}(\omega)$. This spectral function is introduced to set the magnitude of the zero-point radiation. Its value in the classical form is given in the literature (e.g., [22]) as

$$h_{zp}^2(\omega) = \frac{\hbar\omega}{2\pi^2}. \tag{B.5}$$

However, its value in the QED formulation is not found in the literature. Boyer in his pioneering paper on the ZPF in the QED formulation [26] uses the value $H_{zp}^2(\omega) = \hbar\omega/4\pi^2$ without explicitly mentioning any justification. It will be shown below that the magnitude of this spectral function in the QED formulation is indeed

$$H_{zp}^2(\omega) = \frac{\hbar\omega}{4\pi^2}. \tag{B.6}$$

B.2 Determination of the Energy Density

We first determine the energy density of the zero-point field in both the classical and the quantum formulations. In the classical SED, the *average* energy density can be found by calculating

$$\begin{aligned}\langle U(\mathbf{x},t)\rangle &= \frac{1}{8\pi}\left\langle \mathbf{E}^2(\mathbf{x},t) + \mathbf{B}^2(\mathbf{x},t)\right\rangle \\ &= \frac{2}{8\pi}\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3k_1\int d^3k_2\hat{\epsilon}(\mathbf{k}_1,\lambda_1)\cdot\hat{\epsilon}(\mathbf{k}_2,\lambda_2)h_{zp}(\omega_1)h_{zp}(\omega_2)\frac{1}{2}\delta_{\lambda_1\lambda_2}\delta^3\left(\mathbf{k}_1-\mathbf{k}_2\right).\end{aligned} \tag{B.7}$$

The factor of two in the second equality comes from the assumed equal contributions from the electric and magnetic components, and the two delta functions at the end come from the average value of the cosine function, namely,

$$\langle\cos\theta(\mathbf{k}_1,\lambda_1)\cos\theta(\mathbf{k}_2,\lambda_2)\rangle = \frac{1}{2}\delta_{\lambda_1\lambda_2}\delta^3\left(\mathbf{k}_1-\mathbf{k}_2\right). \tag{B.8}$$

Considering again equal contributions from each of the polarization states, $\lambda = 1$ and 2, the Eq.(B.7) becomes,

$$\begin{aligned}\langle U\rangle &= \frac{1}{8\pi}\sum_{\lambda=1}^{2}\int d^3k\hat{\epsilon}(\mathbf{k},\lambda)\cdot\hat{\epsilon}(\mathbf{k},\lambda)h_{zp}^2(\omega)\\ &= \frac{1}{4\pi}\int d^3kh_{zp}^2(\omega)\\ &= \frac{1}{4\pi c^3}\int d\omega d\Omega\omega^2 h_{zp}^2(\omega),\end{aligned} \tag{B.9}$$

where the variable of integration was changed from k to ω using the relation $\omega = k/c$. It is to be noted here that the expression above is independent of any space or time coordinates, which shows the *homogeneity* property of the ZPF energy density per frequency mode.

Since the energy density can also be written as

$$< U >= \int \rho(\omega)d\omega d\Omega, \tag{B.10}$$

we compare this equation with Eq.(B.9) and identify the classical spectral energy density per solid angle Ω in the angular frequency interval between ω and $\omega + d\omega$ as

$$\rho_{cl}(\omega)d\omega = \frac{\omega^2}{4\pi c^3}h_{zp}^2(\omega)d\omega. \tag{B.11}$$

where the subscript *cl* stands for classical, to distinguish this from the QED case.

Now we find the expression for the energy density in the QED formulation. Following the same procedures as the SED case, we obtain

$$
\begin{aligned}
\langle 0 | U(\mathbf{x}, t) | 0 \rangle &= \frac{1}{8\pi} \left\langle 0 \left| \overline{\mathbf{E}}^2(\mathbf{x}, t) + \overline{\mathbf{B}}^2(\mathbf{x}, t) \right| 0 \right\rangle \\
&= \frac{2}{8\pi} \sum_{\lambda_1=1}^{2} \sum_{\lambda_2=1}^{2} \int d^3k_1 \int d^3k_2 \hat{\epsilon}(\mathbf{k}_1, \lambda_1) \cdot \hat{\epsilon}(\mathbf{k}_2, \lambda_2) \\
&\quad \times H_{zp}(\omega_1) H_{zp}(\omega_2) e^{i\Theta_1} e^{-i\Theta_2} \left\langle 0 \left| \alpha(\mathbf{k}_1, \lambda_1) \alpha^\dagger(\mathbf{k}_2, \lambda_2) \right| 0 \right\rangle \\
&= \frac{1}{4\pi} \sum_{\lambda_1=1}^{2} \sum_{\lambda_2=1}^{2} \int d^3k_1 \int d^3k_2 \hat{\epsilon}(\mathbf{k}_1, \lambda_1) \cdot \hat{\epsilon}(\mathbf{k}_2, \lambda_2) \\
&\quad \times H_{zp}(\omega_1) H_{zp}(\omega_2) e^{i\Theta_1} e^{-i\Theta_2} \delta_{\lambda_1 \lambda_2} \delta^3(\mathbf{k}_1 - \mathbf{k}_2)
\end{aligned} \tag{B.12}
$$

where

$$\Theta_1(\mathbf{k}_1) = \mathbf{k}_1 \cdot \mathbf{x} - \omega_1 t \tag{B.13}$$

$$\Theta_2(\mathbf{k}_2) = \mathbf{k}_2 \cdot \mathbf{x} - \omega_2 t \tag{B.14}$$

and the expectation values

$$\left\langle 0 \left| \alpha(\mathbf{k}, \lambda) \alpha(\mathbf{k}', \lambda') \right| 0 \right\rangle = \left\langle 0 \left| \alpha^\dagger(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') \right| 0 \right\rangle = 0 \tag{B.15}$$

$$\left\langle 0 \left| \alpha(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') \right| 0 \right\rangle = \delta_{\lambda,\lambda'} \delta^3(\mathbf{k} - \mathbf{k}') \tag{B.16}$$

$$\left\langle 0 \left| \alpha^\dagger(\mathbf{k}, \lambda) \alpha(\mathbf{k}', \lambda') \right| 0 \right\rangle = 0 \tag{B.17}$$

were used. After integrating over the k-sphere, and taking again equal contributions from each polarization index $\lambda = 1$ and 2 into account, Eq.(B.12) simplifies to

$$
\begin{aligned}
\langle 0 | U(\mathbf{x}, t) | 0 \rangle &= \frac{1}{4\pi} \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}(\mathbf{k}, \lambda) \cdot \hat{\epsilon}(\mathbf{k}, \lambda) H_{zp}^2(\omega) e^{i\Theta} e^{-i\Theta} \\
&= \frac{2}{4\pi} \int d^3k H_{zp}^2(\omega) \\
&= \frac{1}{2\pi c^3} \int d\omega d\Omega \omega^2 H_{zp}^2(\omega).
\end{aligned} \tag{B.18}
$$

Comparing this with the Eq.(B.10), we can identify the energy density per solid angle $d\Omega$ in the bandwidth $d\omega$ in QED formulation as

$$\rho_{QE}(\omega)d\omega = \frac{2\omega^2}{4\pi c^3} H_{zp}^2(\omega)d\omega. \tag{B.19}$$

B.3 The Density of States

The energy density per bandwidth $d\omega$ determined in the previous section can also be expressed in terms of the density of states. This technique is a standard one and can be found in many textbooks. The treatment below basically follows that by de la Pena and Cetto [19] and Louisell [27].

Let $d\mathcal{N}(\omega)$ represent the number of ZPF modes of frequency ω that can be accommodated inside a box of length L. Then the total energy inside the box can be expressed as a product of $d\mathcal{N}(\omega)$ and the energy of each single mode, which leads to the expression for the ZPF energy density:

$$\rho(\omega)d\omega = d\mathcal{N}(\omega)\varepsilon_0(\omega)/V \tag{B.20}$$

where $\varepsilon_0(\omega)$ is the energy of the ZPF spectrum per mode, $(1/2)\hbar\omega$ and $V = L^3$ is the volume of the box. The density of states can be obtained from purely geometrical considerations as follows.

The number of normal modes in a given frequency range inside a small element of volume $dl_1 dl_2 dl_3$ is

$$d\mathcal{N} = 2dl_1 dl_2 dl_3, \tag{B.21}$$

where the factor of two comes from the presence of two polarization states in each direction, and the volume increment $dl_i,\ i = 1, 2, 3$ can be found from

$$\mathbf{k} = \frac{2\pi}{L}(l_1\hat{\imath} + l_2\hat{\jmath} + l_3\hat{k}). \tag{B.22}$$

The set of numbers (l_1, l_2, l_3) represent the number of modes that can be accommodated on each side. Using Eq.(B.21) and Eq.(B.22), we can obtain

$$d\mathcal{N} = 2\left(\frac{L}{2\pi}\right)^3 dk_x dk_y dk_z = 2\left(\frac{L}{2\pi}\right)^3 d^3k = 2\left(\frac{L}{2\pi}\right)^3 k^2 dk d\Omega. \tag{B.23}$$

Changing the integration variable to ω via the relation

$$k^2 dk = \frac{\omega^2}{c^3} d\omega, \tag{B.24}$$

Eq.(B.23) becomes

$$d\mathcal{N} = 2\left(\frac{L}{2\pi c}\right)^3 \omega^2 d\omega d\Omega. \tag{B.25}$$

Note that, since this density of modes was derived from purely geometrical point of view, this expression stays the same regardless of whether the fields are treated classically or quantum electrodynamically.

B.4 Magnitude of Spectral Function

Substituting the density of modes obtained above divided by the solid angle $d\Omega$ into Eq.(B.20), the expression for the energy density may be determined as

$$\begin{aligned}
\rho(\omega)d\omega &= d\mathcal{N}(\omega)\varepsilon_0(\omega)/V \\
&= 2\left(\frac{L}{2\pi c}\right)^3 \omega^2 d\omega \left(\frac{\frac{1}{2}\hbar\omega}{L^3}\right) \\
&= \left(\frac{\hbar\omega^3}{8\pi^3 c^3}\right) d\omega
\end{aligned} \tag{B.26}$$

where $\rho(\omega)$ is given by Eq.(B.11) and Eq.(B.19) in the classical and quantum cases respectively. Comparing these values for $\rho(\omega)$ with the expression above, the spectral functions can be determined as

$$\rho_{cl}(\omega) = \frac{\omega^2}{4\pi c^3} h_{zp}^2(\omega) = \frac{\hbar\omega^3}{8\pi^3 c^3} \quad \Longrightarrow \quad h_{zp}^2(\omega) = \frac{\hbar\omega}{2\pi^2} \tag{B.27}$$

in the classical case. This is the same value as given in the SED literatures. In the QED formulation, however, the magnitude of the spectral function is found to be

$$\rho_{QE}(\omega) = \frac{2\omega^2}{4\pi c^3} H_{zp}^2(\omega) = \frac{\hbar\omega^3}{8\pi^3 c^3} \quad \Longrightarrow \quad H_{zp}^2(\omega) = \frac{\hbar\omega}{4\pi^2}, \tag{B.28}$$

confirming that the scale of the spectral function in the QED formulation differs from that of the classical case by a factor of $1/2$.

C Detailed Calculations of Vacuum Expectation Values: Momentum Flux Approach

C.1 Overview

In this section, detailed calculations for each component of the vacuum expectation values will be shown. The ZPF in the instantaneous comoving frame was found in the Chapter 4 to be

$$\begin{aligned}
\mathbf{E}^{zp}(0,\tau) = \sum_{\lambda=1}^{2}\int d^3k \Big\{ & \hat{x}\hat{\epsilon}_x + \hat{y}\cosh\left(\frac{a\tau}{c}\right)\left[\hat{\epsilon}_y - \tanh\left(\frac{a\tau}{c}\right)(\hat{k}\times\hat{\epsilon})_z\right] \\
& + \hat{z}\cosh\left(\frac{a\tau}{c}\right)\left[\hat{\epsilon}_z + \tanh\left(\frac{a\tau}{c}\right)(\hat{k}\times\hat{\epsilon})_y\right]\Big\} H_{zp}(\omega) \\
& \times\left\{\alpha\left(\mathbf{k},\lambda\right)\exp\left[i\Theta\right] + \alpha^{\dagger}\left(\mathbf{k},\lambda\right)\exp\left[-i\Theta\right]\right\}
\end{aligned} \tag{C.1}$$

$$\begin{aligned}
\mathbf{B}^{zp}(0,\tau) = \sum_{\lambda=1}^{2}\int d^3k \Big\{ & \hat{x}(\hat{k}\times\hat{\epsilon})_x + \hat{y}\cosh\left(\frac{a\tau}{c}\right)\left[(\hat{k}\times\hat{\epsilon})_y + \tanh\left(\frac{a\tau}{c}\right)\hat{\epsilon}_z\right] \\
& + \hat{z}\cosh\left(\frac{a\tau}{c}\right)\left[(\hat{k}\times\hat{\epsilon})_z - \tanh\left(\frac{a\tau}{c}\right)\hat{\epsilon}_y\right]\Big\} H_{zp}(\omega) \\
& \times\left\{\alpha\left(\mathbf{k},\lambda\right)\exp\left[i\Theta\right] + \alpha^{\dagger}\left(\mathbf{k},\lambda\right)\exp\left[-i\Theta\right]\right\}
\end{aligned} \tag{C.2}$$

where

$$\Theta = k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right) \tag{C.3}$$

and

$$\Theta' = k_x'\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega'\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right). \tag{C.4}$$

Now we are going to evaluate the vacuum expectation values of the ZPF Poynting vector, $\left\langle 0\left|E_iB_j\right|0\right\rangle$, where E_i is the i-component (x, y, or z) of the zero-point electric field. The calculations for each of the nine terms are shown below.

C.2 Evaluation of Each Component

C.2.1 $\langle 0 | E_x B_x | 0 \rangle$

In order to evaluate the component $\langle 0 | E_x B_x | 0 \rangle$, the product of the x-components of the ZPF operators (C.1) and (C.2) is formed first and we obtain

$$\begin{aligned}\langle 0 | E_x B_x | 0 \rangle = \sum_{\lambda=1}^{2} \sum_{\lambda'=1}^{2} \int d^3k \int d^3k' \hat{\epsilon}_x (\hat{k}' \times \hat{\epsilon}')_x H_{zp}^2(\omega) \\ \left\langle 0 \left| \left\{ \alpha (\mathbf{k}, \lambda) \exp[i\Theta] + \alpha^{\dagger} (\mathbf{k}, \lambda) \exp[-i\Theta] \right\} \right.\right. \\ \times \left. \left. \left\{ \alpha (\mathbf{k}', \lambda') \exp[i\Theta'] + \alpha^{\dagger} (\mathbf{k}', \lambda') \exp[-i\Theta'] \right\} \right| 0 \right\rangle,\end{aligned} \tag{C.5}$$

where

$$\Theta = k_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right) \tag{C.6}$$

and

$$\Theta' = k_x' \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega' \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right). \tag{C.7}$$

The expression above has four terms. However, only the term proportional to $\left\langle 0 \left| \alpha (\mathbf{k}, \lambda) , \alpha^{\dagger} (\mathbf{k}', \lambda') \right| 0 \right\rangle$ remains as in (C.10), and the above expression is simplified to

$$\begin{aligned}\langle 0 | E_x B_x | 0 \rangle = \sum_{\lambda=1}^{2} \sum_{\lambda'=1}^{2} \int d^3k \int d^3k' \hat{\epsilon}_x (\hat{k}' \times \hat{\epsilon}')_x H_{zp}^2(\omega) \\ \times \left\langle 0 \left| \alpha (\mathbf{k}, \lambda) , \alpha^{\dagger} (\mathbf{k}', \lambda') \right| 0 \right\rangle \exp\left[i\Theta(\mathbf{k})\right] \exp\left[-i\Theta'(\mathbf{k}')\right],\end{aligned} \tag{C.8}$$

with the use of the expectation values

$$\left\langle 0 \left| \alpha (\mathbf{k}, \lambda) , \alpha (\mathbf{k}', \lambda') \right| 0 \right\rangle = \left\langle 0 \left| \alpha^{\dagger} (\mathbf{k}, \lambda) , \alpha^{\dagger} (\mathbf{k}', \lambda') \right| 0 \right\rangle = 0 \tag{C.9}$$

$$\left\langle 0 \left| \alpha (\mathbf{k}, \lambda) , \alpha^{\dagger} (\mathbf{k}', \lambda') \right| 0 \right\rangle = \delta_{\lambda,\lambda'} \delta^3 (\mathbf{k} - \mathbf{k}') \tag{C.10}$$

$$\left\langle 0 \left| \alpha^{\dagger} (\mathbf{k}, \lambda) , \alpha (\mathbf{k}', \lambda') \right| 0 \right\rangle = 0. \tag{C.11}$$

Since the term in the second line in (C.8) is $\delta_{\lambda,\lambda'}\delta^3(\mathbf{k}-\mathbf{k}')\exp[i\Theta(\mathbf{k})]\exp[-i\Theta'(\mathbf{k}')]$, Eq.(C.8) becomes

$$\langle 0|E_xB_x|0\rangle = \sum_{\lambda=1}^{2}\sum_{\lambda'=1}^{2}\int d^3k\int d^3k'\hat{\epsilon}_x(\hat{k}'\times\hat{\epsilon}')_x H_{zp}^2(\omega)$$
$$\times\delta_{\lambda,\lambda'}\delta^3(\mathbf{k}-\mathbf{k}')\exp[i\Theta(\mathbf{k})]\exp[-i\Theta'(\mathbf{k}')]\,, \tag{C.12}$$

which, after one integration over the k-sphere, reduces to

$$\langle 0|E_xB_x|0\rangle = \sum_{\lambda=1}^{2}\int d^3k\hat{\epsilon}_x(\hat{k}\times\hat{\epsilon})_x H_{zp}^2(\omega). \tag{C.13}$$

With a use of the polarization formula, we find that

$$\sum_{\lambda=1}^{2}\hat{\epsilon}_x(\hat{k}\times\hat{\epsilon})_x = \sum_{k=x,y,z}\varepsilon_{iik}\hat{k}_k = 0, \tag{C.14}$$

and after substituting this result into the equation above, it is concluded that $\langle 0|E_xB_x|0\rangle = 0$.

C.2.2 $\left\langle 0\left|E_xB_y\right|0\right\rangle$

$\left\langle 0\left|E_xB_y\right|0\right\rangle$ can also be evaluated in the similar manner as $\langle 0|E_xB_x|0\rangle$. That is, the product of the $x-$component of the electric field and the $y-$component of the magnetic field is formed, which involves two sums and two integrals as in (C.5). Then, the expectation value is taken as before, leaving

$$\left\langle 0\left|E_xB_y\right|0\right\rangle = \sum_{\lambda=1}^{2}\int d^3kH_{zp}^2(\omega)\hat{\epsilon}_x\cosh\frac{a\tau}{c}\left[(\hat{k}\times\hat{\epsilon})_y+\tanh\frac{a\tau}{c}\hat{\epsilon}_z\right] \tag{C.15}$$

In order to evaluate this equation, we use the following polarization formulae again:

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_y = \hat{k}_z, \tag{C.16}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x \hat{\epsilon}_y = -\hat{k}_x \hat{k}_z, \tag{C.17}$$

and obtain

$$\left\langle 0 \left| E_x B_y \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \left[\cosh \frac{a\tau}{c} \hat{k}_z - \sinh \frac{a\tau}{c} \hat{k}_x \hat{k}_z \right]. \tag{C.18}$$

The above expression can be evaluated by integrating over the $k-$sphere using the relation,

$$\int d^3k = \int k^2 dk \int d\Omega = \int k^2 dk \int \sin\theta d\theta \int d\phi. \tag{C.19}$$

On applying this relation, the first term including the $\hat{k}_z$ term gives

$$\int d^3k \hat{k}_z = \int k^2 dk \int \sin\theta \cos\theta d\theta \int d\phi = 0, \tag{C.20}$$

since the angular integration $\int \sin\theta \cos\theta d\theta$ is zero. Similarly, the second term involving the $\hat{k}_x \hat{k}_z$ reduces to

$$\int d^3k \hat{k}_x \hat{k}_z = \int k^2 dk \int \sin^2\theta \cos\theta d\theta \int \cos\phi d\phi = 0, \tag{C.21}$$

due to the vanishing azimuthal integration $\int \cos\phi d\phi$, and it is concluded that $\left\langle 0 \left| E_x B_y \right| 0 \right\rangle$ is also zero.

C.2.3 $\langle 0 | E_x B_z | 0 \rangle$

We continue the same analysis on other components. For $\langle 0 | E_x B_z | 0 \rangle$, we obtain

$$\langle 0 | E_x B_z | 0 \rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \hat{\epsilon}_x \left[\cosh \frac{a\tau}{c} (\hat{k} \times \hat{\epsilon})_z - \sinh \frac{a\tau}{c} \hat{\epsilon}_y \right]. \tag{C.22}$$

Again, we make use of the polarization equations,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_z = -\hat{k}_y \tag{C.23}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x \hat{\epsilon}_y = -\hat{k}_x \hat{k}_y, \tag{C.24}$$

and obtain

$$\langle 0 | E_x B_z | 0 \rangle = \sum_{\lambda=1}^{2} \int d^3 k H_{zp}^2(\omega) \left[\sinh \frac{a\tau}{c} \hat{k}_y - \cosh \frac{a\tau}{c} \hat{k}_x \hat{k}_y \right]. \tag{C.25}$$

After the $k-$sphere integration, the two terms in the above equation reduce to

$$\int d^3 k \hat{k}_y = \int k^2 dk \int \sin^2\theta d\theta \int \sin\phi d\phi = 0, \tag{C.26}$$

and

$$\int d^3 k \hat{k}_x \hat{k}_y = \int k^2 dk \int \sin^3\theta d\theta \int \sin\phi \cos\phi d\phi = 0, \tag{C.27}$$

both due to the vanishing azimuthal integrations, and it is concluded that $\langle 0 | E_x B_z | 0 \rangle$ is also zero. This is actually a well expected result. Since it is assumed that the object is accelerating along the $x-$axis, there should exist a symmetry about this direction, and the value of $\langle 0 | E_x B_z | 0 \rangle$ should be the same as that of $\left\langle 0 \left| E_x B_y \right| 0 \right\rangle$.

C.2.4 $\left\langle 0 \left| E_y B_x \right| 0 \right\rangle$

$$\left\langle 0 \left| E_y B_x \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3 k H_{zp}^2(\omega) \left[\cosh \frac{a\tau}{c} \hat{\epsilon}_y - \sinh \frac{a\tau}{c} (\hat{k} \times \hat{\epsilon})_z \right] (\hat{k} \times \hat{\epsilon})_x. \tag{C.28}$$

We make use of the following polarization equations,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y (\hat{k} \times \hat{\epsilon})_x = -\hat{k}_z, \tag{C.29}$$

$$\sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_z (\hat{k} \times \hat{\epsilon})_x = -\hat{k}_x \hat{k}_z. \tag{C.30}$$

Since, as we have seen before, both terms disappear after the integrations,

$$\int d^3k\hat{k}_z = 0, \tag{C.31}$$

and

$$\int d^3k\hat{k}_x\hat{k}_z = 0, \tag{C.32}$$

it is concluded that

$$\left\langle 0\left|E_yB_x\right|0\right\rangle = 0. \tag{C.33}$$

C.2.5 $\left\langle 0\left|E_yB_y\right|0\right\rangle$

$$\begin{aligned}\left\langle 0\left|E_yB_y\right|0\right\rangle = &\sum_{\lambda=1}^{2}\int d^3kH_{zp}^2(\omega)\\ &\times\left[\cosh\frac{a\tau}{c}\hat{\epsilon}_y - \sinh\frac{a\tau}{c}(\hat{k}\times\hat{\epsilon})_z\right]\left[\cosh\frac{a\tau}{c}(\hat{k}\times\hat{\epsilon})_y + \sinh\frac{a\tau}{c}\hat{\epsilon}_z\right].\end{aligned} \tag{C.34}$$

This equation has four terms, and each of them are to be obtained with the use of the following polarization equations:

$$\begin{aligned}\sum_{\lambda=1}^{2}\hat{\epsilon}_y(\hat{k}\times\hat{\epsilon})_y &= 0\\ \sum_{\lambda=1}^{2}(\hat{k}\times\hat{\epsilon})_z(\hat{k}\times\hat{\epsilon})_y &= -\hat{k}_y\hat{k}_z\\ \sum_{\lambda=1}^{2}\hat{\epsilon}_y\hat{\epsilon}_z &= -\hat{k}_y\hat{k}_z\\ \sum_{\lambda=1}^{2}(\hat{k}\times\hat{\epsilon})_z\hat{\epsilon}_z &= 0.\end{aligned}$$

With these results, it is shown that,

$$\left\langle 0\left|E_yB_y\right|0\right\rangle = \sum_{\lambda=1}^{2}\int d^3kH_{zp}^2(\omega)\left[\cosh\frac{a\tau}{c}\sinh\frac{a\tau}{c}\right](\hat{k}_y\hat{k}_z - \hat{k}_y\hat{k}_z) = 0. \tag{C.35}$$

C.2.6 $\left\langle 0 \left| E_y B_z \right| 0 \right\rangle$

$$\left\langle 0 \left| E_y B_z \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \times \left[\cosh \frac{a\tau}{c} \hat{\epsilon}_y - \sinh \frac{a\tau}{c} (\hat{k} \times \hat{\epsilon})_z\right] \left[\cosh \frac{a\tau}{c} (\hat{k} \times \hat{\epsilon})_z - \sinh \frac{a\tau}{c} \hat{\epsilon}_y\right]. \tag{C.36}$$

This equation also has four terms. We evaluate these using the following polarization equations,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y (\hat{k} \times \hat{\epsilon})_z = \hat{k}_x$$
$$\sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_z (\hat{k} \times \hat{\epsilon})_z = \hat{k}_x^2 + \hat{k}_y^2 = 1 - \hat{k}_z^2$$
$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y^2 = 1 - \hat{k}_y^2.$$

Combining the above results, it is shown that,

$$\begin{aligned}\left\langle 0 \left| E_y B_z \right| 0 \right\rangle &= \int d^3k H_{zp}^2(\omega) \\ &\quad \times \left\{\left[\cosh^2 \frac{a\tau}{c} + \sinh^2 \frac{a\tau}{c}\right] \hat{k}_x - \cosh \frac{a\tau}{c} \sinh \frac{a\tau}{c} \left[(1 - \hat{k}_z^2) + (1 - \hat{k}_y^2)\right]\right\} \\ &= \int d^3k H_{zp}^2(\omega) \left\{\left[\cosh^2 \frac{a\tau}{c} + \sinh^2 \frac{a\tau}{c}\right] \hat{k}_x - \cosh \frac{a\tau}{c} \sinh \frac{a\tau}{c} \left[2 - \hat{k}_y^2 - \hat{k}_z^2\right]\right\}.\end{aligned} \tag{C.37}$$

Using the relation,

$$1 = \hat{k}_x^2 + \hat{k}_y^2 + \hat{k}_z^2, \tag{C.38}$$

the above expectation value can be simplified as,

$$\left\langle 0 \left| E_y B_z \right| 0 \right\rangle = \int d^3k H_{zp}^2(\omega) \left\{\left[\cosh^2 \frac{a\tau}{c} + \sinh^2 \frac{a\tau}{c}\right] \hat{k}_x - \cosh \frac{a\tau}{c} \sinh \frac{a\tau}{c} \left[1 + \hat{k}_x^2\right]\right\}. \tag{C.39}$$

The first term of this equation is zero since $\int d^3k\hat{k}_x = 0$, and we obtain,

$$\left\langle 0 \left| E_y B_z \right| 0 \right\rangle = -\int d^3k H_{zp}^2(\omega) \cosh\frac{a\tau}{c} \sinh\frac{a\tau}{c} \left[1 + \hat{k}_x^2\right]. \tag{C.40}$$

After substituting

$$H_{zp}^2(\omega) = \frac{\hbar\omega}{4\pi^2}, \tag{C.41}$$

$$dk = d\omega/c, \tag{C.42}$$

and,

$$\sinh\theta\cosh\theta = \frac{1}{2}\sinh(2\theta), \tag{C.43}$$

we obtain,

$$\left\langle 0 \left| E_y B_z \right| 0 \right\rangle = -\frac{1}{2}\sinh\frac{2a\tau}{c} \int d\omega \frac{\hbar\omega^3}{4\pi^2 c^3} \int d\Omega \left(1 + \hat{k}_x^2\right). \tag{C.44}$$

The angle integrations in the above equation gives

$$\int d\Omega = \int \sin\theta d\theta d\phi = 4\pi$$
$$\int \hat{k}_x^2 d\Omega = \int \sin^3\theta d\theta \int \cos\phi d\phi = \frac{4\pi}{3}$$

after minimal algebra, and the expectation value is found to be

$$\left\langle 0 \left| E_y B_z \right| 0 \right\rangle = -\frac{4\pi}{3}\sinh\frac{2a\tau}{c} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \tag{C.45}$$

C.2.7 $\langle 0 | E_z B_x | 0 \rangle$

Due to symmetry about the direction of acceleration, i.e., the x−axis, the value of

$\langle 0 | E_z B_x | 0 \rangle$ should be the same as that of $\left\langle 0 \left| E_y B_x \right| 0 \right\rangle$. Therefore, it is concluded that

$$\langle 0 | E_z B_x | 0 \rangle = 0. \tag{C.46}$$

C.2.8 $\left\langle 0 \left| E_z B_y \right| 0 \right\rangle$

The value of $\left\langle 0 \left| E_z B_y \right| 0 \right\rangle$ should be proportional to that of $\left\langle 0 \left| E_y B_z \right| 0 \right\rangle$, due to the symmetry around the $x-$axis. This is found from the equation

$$\begin{aligned}\left\langle 0 \left| E_z B_y \right| 0 \right\rangle = &\sum_{\lambda=1}^{2} \int d^3 k H_{zp}^2(\omega) \\ &\times \left[\cosh \frac{a\tau}{c} \hat{\epsilon}_z + \sinh \frac{a\tau}{c} (\hat{k} \times \hat{\epsilon})_y \right] \left[\cosh \frac{a\tau}{c} (\hat{k} \times \hat{\epsilon})_y + \sinh \frac{a\tau}{c} \hat{\epsilon}_z \right]\end{aligned} \tag{C.47}$$

with the polarization equations,

$$\begin{aligned}\sum_{\lambda=1}^{2} \hat{\epsilon}_z (\hat{k} \times \hat{\epsilon})_y &= -\hat{k}_x \\ \sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_y (\hat{k} \times \hat{\epsilon})_y &= \hat{k}_x^2 + \hat{k}_z^2 = 1 - \hat{k}_y^2 \\ \sum_{\lambda=1}^{2} \hat{\epsilon}_z^2 &= 1 - \hat{k}_z^2.\end{aligned}$$

Combining the above results, it is shown that,

$$\begin{aligned}\left\langle 0 \left| E_z B_y \right| 0 \right\rangle = &\int d^3 k H_{zp}^2(\omega) \\ &\times \left\{ - \left[\cosh^2 \frac{a\tau}{c} + \sinh^2 \frac{a\tau}{c} \right] \hat{k}_x + \cosh \frac{a\tau}{c} \sinh \frac{a\tau}{c} \left[(1 - \hat{k}_y^2) + (1 - \hat{k}_z^2) \right] \right\}.\end{aligned} \tag{C.48}$$

Since the angle integration for the first term is zero as we have seen in the case of $\left\langle 0 \left| E_y B_z \right| 0 \right\rangle$, we have

$$\left\langle 0 \left| E_z B_y \right| 0 \right\rangle = \int d^3 k H_{zp}^2(\omega) \cosh \frac{a\tau}{c} \sinh \frac{a\tau}{c} \left[1 + \hat{k}_x^2 \right]. \tag{C.49}$$

It is found that this is the same expression as the Eq(C.40) for $\left\langle 0 \left| E_y B_z \right| 0 \right\rangle$, except the absence of the negative sign. Therefore, we conclude that

$$\left\langle 0 \left| E_z B_y \right| 0 \right\rangle = \frac{4\pi}{3} \sinh \frac{2a\tau}{c} \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega, \tag{C.50}$$

which is also the same value as $\left\langle 0 \left| E_y B_z \right| 0 \right\rangle$ (Eq.C.45), except the opposite sign.

C.2.9 $\langle 0 | E_z B_z | 0 \rangle$

Due to the cylindrical symmetry around the x–axis, the value of $\langle 0 | E_z B_z | 0 \rangle$ should vanish as in the case of $\left\langle 0 \left| E_y B_y \right| 0 \right\rangle$. Hence

$$\langle 0 | E_z B_z | 0 \rangle = 0. \tag{C.51}$$

D Detailed Calculations of Vacuum Expectation Values: Momentum Content Approach

D.1 Overview

Detailed calculations for each component of the vacuum expectation values will be shown in this section. The ZPF in the laboratory inertial frame I_* expressed in terms of the object's instantaneous comoving frame ZPF components was obtained in the Chapter4 as shown below.

$$\begin{aligned}
\mathbf{E}_*^{zp}(0,\tau) = \sum_{\lambda=1}^{2}\int d^3k \Big\{&\hat{x}\hat{\epsilon}_x + \hat{y}\gamma_\tau\left[\hat{\epsilon}_y + \beta_\tau(\hat{k}\times\hat{\epsilon})_z\right] \\
&+ \hat{z}\gamma_\tau\left[\hat{\epsilon}_z - \beta_\tau(\hat{k}\times\hat{\epsilon})_y\right]\Big\} H_{zp}(\omega) \\
&\times\left\{\alpha(\mathbf{k},\lambda)\exp[i\Theta] + \alpha^\dagger(\mathbf{k},\lambda)\exp[-i\Theta]\right\} \qquad \text{(D.1)}\\
\mathbf{B}_*^{zp}(0,\tau) = \sum_{\lambda=1}^{2}\int d^3k \Big\{&\hat{x}(\hat{k}\times\hat{\epsilon})_x + \hat{y}\gamma_\tau\left[(\hat{k}\times\hat{\epsilon})_y - \beta_\tau\hat{\epsilon}_z\right] \\
&+ \hat{z}\gamma_\tau\left[(\hat{k}\times\hat{\epsilon})_z + \beta_\tau\hat{\epsilon}_y\right]\Big\} H_{zp}(\omega) \\
&\times\left\{\alpha(\mathbf{k},\lambda)\exp[i\Theta] + \alpha^\dagger(\mathbf{k},\lambda)\exp[-i\Theta]\right\} \qquad \text{(D.2)}
\end{aligned}$$

where

$$\Theta = k_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right) \qquad \text{(D.3)}$$

and

$$\Theta' = k'_x\frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) - \omega'\frac{c}{a}\sinh\left(\frac{a\tau}{c}\right). \qquad \text{(D.4)}$$

In order to evaluate the expectation values $\left\langle 0\left|E_{i*}B_{j*}\right|0\right\rangle$, the i-th component of the Zero-Point electric field (D.1) and the j-th component of the magnetic field (D.2) are multiplied together. The detailed calculations for each of the nine terms are shown below.

D.2 Evaluation of Each Component

D.2.1 $\langle 0 | E_{x*} B_{x*} | 0 \rangle$

The vacuum expectation value $\langle 0 | E_{x*} B_{x*} | 0 \rangle$ may be evaluated by forming the product of the x-components of the ZPF operators (D.1) and (D.2) and by performing several integrations. The integrations has to be done as mentioned before in the object's instantaneous rest frame I_τ. The product of the two field components are given by

$$\begin{aligned} \langle 0 | E_{x*} B_{x*} | 0 \rangle = & \sum_{\lambda=1}^{2} \sum_{\lambda'=1}^{2} \int d^3k \int d^3k' \hat{\epsilon}_x (\hat{k}' \times \hat{\epsilon}')_x H_{zp}(\omega) H_{zp(\omega')} \\ & \left\langle 0 \left| \left\{ \alpha(\mathbf{k}, \lambda) \exp[i\Theta] + \alpha^\dagger(\mathbf{k}, \lambda) \exp[-i\Theta] \right\} \right.\right. \\ & \times \left.\left. \left\{ \alpha(\mathbf{k}', \lambda') \exp[i\Theta'] + \alpha^\dagger(\mathbf{k}', \lambda') \exp[-i\Theta'] \right\} \right| 0 \right\rangle, \end{aligned} \tag{D.5}$$

where

$$\Theta = k_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right) \tag{D.6}$$

and

$$\Theta' = k'_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega' \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right). \tag{D.7}$$

With the help of the expectation value relationships

$$\left\langle 0 \left| \alpha(\mathbf{k}, \lambda), \alpha(\mathbf{k}', \lambda') \right| 0 \right\rangle = \left\langle 0 \left| \alpha^\dagger(\mathbf{k}, \lambda), \alpha^\dagger(\mathbf{k}', \lambda') \right| 0 \right\rangle = 0 \tag{D.8}$$

$$\left\langle 0 \left| \alpha(\mathbf{k}, \lambda), \alpha^\dagger(\mathbf{k}', \lambda') \right| 0 \right\rangle = \delta_{\lambda,\lambda'} \delta^3(\mathbf{k} - \mathbf{k}') \tag{D.9}$$

$$\left\langle 0 \left| \alpha^\dagger(\mathbf{k}, \lambda), \alpha(\mathbf{k}', \lambda') \right| 0 \right\rangle = 0, \tag{D.10}$$

we can immediately understand that among the four terms above, only the one proportional to $\left\langle 0 \left| \alpha\left(\mathbf{k}, \lambda\right), \alpha^{\dagger}\left(\mathbf{k}', \lambda'\right)\right| 0 \right\rangle$ remains and the above expression simplifies to

$$\begin{aligned}
\langle 0 \left| E_{x*} B_{x*} \right| 0 \rangle = & \sum_{\lambda=1}^{2} \sum_{\lambda'=1}^{2} \int d^3k \int d^3k' \hat{\epsilon}_x (\hat{k}' \times \hat{\epsilon}')_x H_{zp}^2(\omega) \\
& \times \left\langle 0 \left| \alpha\left(\mathbf{k}, \lambda\right), \alpha^{\dagger}\left(\mathbf{k}', \lambda'\right)\right| 0 \right\rangle \exp\left[i\Theta(\mathbf{k})\right] \exp\left[-i\Theta'(\mathbf{k}')\right] \qquad \text{(D.11)} \\
= & \sum_{\lambda=1}^{2} \sum_{\lambda'=1}^{2} \int d^3k \int d^3k' \hat{\epsilon}_x (\hat{k}' \times \hat{\epsilon}')_x H_{zp}^2(\omega) \\
& \times \delta_{\lambda,\lambda'} \delta^3\left(\mathbf{k} - \mathbf{k}'\right) \exp[i\Theta(\mathbf{k})] \exp\left[-i\Theta'(\mathbf{k}')\right], \qquad \text{(D.12)}
\end{aligned}$$

which, after one integration over the k-sphere, reduces to

$$\langle 0 \left| E_{x*} B_{x*} \right| 0 \rangle = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_x H_{zp}^2(\omega). \qquad \text{(D.13)}$$

Using one of the polarization formula,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_x = \sum_{k=x,y,z} \varepsilon_{iik} \hat{k}_k = 0, \qquad \text{(D.14)}$$

and after substituting this result into the equation above, it is concluded that

$$\langle 0 \left| E_x B_x \right| 0 \rangle = 0. \qquad \text{(D.15)}$$

D.2.2 $\left\langle 0 \left| E_{x*} B_{y*} \right| 0 \right\rangle$

The product of the zero-point electric and magnetic field components are given by

$$\left\langle 0 \left| E_{x*} B_{y*} \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \left\{ \hat{\epsilon}_x \gamma_\tau \left[\left(\hat{k} \times \hat{\epsilon} \right)_y - \beta_\tau \hat{\epsilon}_z \right] \right\}, \qquad \text{(D.16)}$$

which can be simplified with the use of the following polarization formulae:

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x (\hat{k} \times \hat{\epsilon})_y = \hat{k}_z, \tag{D.17}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x \hat{\epsilon}_y = -\hat{k}_x \hat{k}_z, \tag{D.18}$$

and the Eq.(D.16) reduces to

$$\left\langle 0 \left| E_{x*} B_{y*} \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \left[\gamma_\tau \hat{k}_z + \gamma_\tau \beta_\tau \hat{k}_x \hat{k}_z \right]. \tag{D.19}$$

Each term of the equation above will be evaluated by integrating over the $k-$sphere,

$$\int d^3k \hat{k}_z = \int k^2 dk \int\int \sin\theta \cos\theta \, d\theta \, d\phi = 2\pi \int k^2 dk \int \sin\theta \cos\theta \, d\theta = 0, \tag{D.20}$$

$$\int d^3k \hat{k}_x \hat{k}_z = \int k^2 dk \int \sin^2\theta \cos\theta \, d\theta \int \cos\phi \, d\phi = 0, \tag{D.21}$$

since the azimuthal integration $\int \cos\phi d\phi$ yields zero. Therefore, we obtain

$$\left\langle 0 \left| E_{x*} B_{y*} \right| 0 \right\rangle = 0. \tag{D.22}$$

D.2.3 $\langle 0 | E_{x*} B_{z*} | 0 \rangle$

The equation to be evaluated would be

$$\langle 0 | E_{x*} B_{z*} | 0 \rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \left\{ \hat{\epsilon}_x \gamma_\tau \left[\left(\hat{k} \times \hat{\epsilon} \right)_z + \beta_\tau \hat{\epsilon}_y \right] \right\}. \tag{D.23}$$

The polarization equations used for this are

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x \left(\hat{k} \times \hat{\epsilon}\right)_z = -\hat{k}_y \tag{D.24}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x \hat{\epsilon}_y = -\hat{k}_x \hat{k}_y, \tag{D.25}$$

and the Eq.(D.23) simplifies to

$$\langle 0 \left| E_{x*} B_{z*} \right| 0 \rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \left[\gamma_\tau \hat{k}_y + \beta_\tau \hat{k}_x \hat{k}_y\right]. \tag{D.26}$$

Again we perform the $k-$sphere integration and obtain

$$\int d^3k \hat{k}_y = \int k^2 dk \int \sin^2\theta \, d\theta \int \sin\phi \, d\phi = 0, \tag{D.27}$$

and

$$\int d^3k \hat{k}_x \hat{k}_y = \int k^2 dk \int \sin^3\theta \, d\theta \int \sin\phi \cos\phi \, d\phi = 0. \tag{D.28}$$

Both integrations are zero due to the vanishing azimuthal integrations, and we obtain again

$$\langle 0 \left| E_{x*} B_{z*} \right| 0 \rangle = 0. \tag{D.29}$$

Since the object is moving in the positive x-direction, there exists a symmetry about x-axis. Therefore, it is reasonable that we have obtained the same values for both $\left\langle 0 \left| E_{x*} B_{y*} \right| 0 \right\rangle$ and $\langle 0 \left| E_{x*} B_{z*} \right| 0 \rangle$.

D.2.4 $\left\langle 0 \left| E_{y*} B_{x*} \right| 0 \right\rangle$

After multiplying the y-component of the ZPF electric field by the x-component of the magnetic field, we obtain

$$\left\langle 0 \left| E_{y*} B_{x*} \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \left\{\gamma_\tau \left[\hat{\epsilon}_y + \beta_\tau \left(\hat{k} \times \hat{\epsilon}\right)_z\right] \left(\hat{k} \times \hat{\epsilon}\right)_x\right\}. \tag{D.30}$$

The use of the following polarization equations,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y (\hat{k} \times \hat{\epsilon})_x = -\hat{k}_z, \tag{D.31}$$

$$\sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_z (\hat{k} \times \hat{\epsilon})_x = -\hat{k}_x \hat{k}_z, \tag{D.32}$$

yields the same k-sphere integration (D.20) and (D.21) already evaluated previously. Hence, we conclude

$$\left\langle 0 \left| E_{y*} B_{x*} \right| 0 \right\rangle = 0. \tag{D.33}$$

D.2.5 $\left\langle 0 \left| E_{y*} B_{y*} \right| 0 \right\rangle$

$$\begin{aligned} \left\langle 0 \left| E_{y*} B_{y*} \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \\ \times \left[\gamma_\tau \hat{\epsilon}_y + \beta_\tau \left(\hat{k} \times \hat{\epsilon} \right)_z \right] \left[\gamma_\tau \left(\hat{k} \times \hat{\epsilon} \right)_y - \beta_\tau \hat{\epsilon}_z \right] \end{aligned} \tag{D.34}$$

Among the four terms in the equation above, two terms immediately reduce to zero upon application of the polarization formula:

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y \left(\hat{k} \times \hat{\epsilon} \right)_y = 0$$

$$\sum_{\lambda=1}^{2} \left(\hat{k} \times \hat{\epsilon} \right)_z \hat{\epsilon}_z = 0,$$

and the Eq.(D.34) simplifies to

$$\left\langle 0 \left| E_{y*} B_{y*} \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \times \gamma_\tau^2 \beta_\tau \left[-\hat{\epsilon}_y \hat{\epsilon}_z + \left(\hat{k} \times \hat{\epsilon} \right)_z \left(\hat{k} \times \hat{\epsilon} \right)_y \right]. \tag{D.35}$$

Applying the polarization equations again, it is found that the non-vanishing two terms have values

$$\sum_{\lambda=1}^{2}\left(\hat{k}\times\hat{\epsilon}\right)_z\left(\hat{k}\times\hat{\epsilon}\right)_y = -\hat{k}_y\hat{k}_z,$$

$$-\sum_{\lambda=1}^{2}\hat{\epsilon}_y\hat{\epsilon}_z = \hat{k}_y\hat{k}_z,$$

which exactly cancels each other. to conclude that

$$\left\langle 0\left|E_{y*}B_{y*}\right|0\right\rangle = 0. \tag{D.36}$$

D.2.6 $\left\langle 0\left|E_{y*}B_{z*}\right|0\right\rangle$

$$\begin{aligned}\left\langle 0\left|E_{y*}B_{z*}\right|0\right\rangle = &\sum_{\lambda=1}^{2}\int d^3kH_{zp}^2(\omega)\\ &\times\gamma_\tau\left[\hat{\epsilon}_y+\beta_\tau\left(\hat{k}\times\hat{\epsilon}\right)_z\right]\gamma_\tau\left[\left(\hat{k}\times\hat{\epsilon}\right)_z+\beta_\tau\hat{\epsilon}_y\right].\end{aligned} \tag{D.37}$$

This equation also has four terms, but two of them yield the same k-sphere integrations we have already evaluated:

$$\sum_{\lambda=1}^{2}\hat{\epsilon}_y\left(\hat{k}\times\hat{\epsilon}\right)_z = \sum_{\lambda=1}^{2}\left(\hat{k}\times\hat{\epsilon}\right)_z\hat{\epsilon}_y = \hat{k}_x,$$

which gives zero after the k integration,

$$\int d^3k\hat{k}_x = \int k^2dk\int\sin^2\theta\,d\theta\int\cos\phi\,d\phi = 0. \tag{D.38}$$

The remaining two terms have values

$$\sum_{\lambda=1}^{2}\left(\hat{k}\times\hat{\epsilon}\right)_z\left(\hat{k}\times\hat{\epsilon}\right)_z = 1-\hat{k}_z^2$$
$$\sum_{\lambda=1}^{2}\hat{\epsilon}_y^2 = 1-\hat{k}_y^2.$$

Then the Eq.(D.37) simplifies to

$$\begin{aligned}\left\langle 0\left|E_{y*}B_{z*}\right|0\right\rangle &= \sum_{\lambda=1}^{2}\int d^3kH_{zp}^2(\omega)\gamma_\tau^2\beta_\tau\left[\hat{\epsilon}_y\hat{\epsilon}_y+\left(\hat{k}\times\hat{\epsilon}\right)_z\left(\hat{k}\times\hat{\epsilon}\right)_z\right] \\ &= \int d^3kH_{zp}^2(\omega)\gamma_\tau^2\beta_\tau\left(2-\hat{k}_y^2-\hat{k}_z^2\right) \\ &= \int d^3kH_{zp}^2(\omega)\gamma_\tau^2\beta_\tau\left(1+\hat{k}_x^2\right)\end{aligned} \tag{D.39}$$

where the relation $1 = \hat{k}_x^2+\hat{k}_y^2+\hat{k}_z^2$ was used in the last step. The k-integrations are given below as

$$\int d^3k = \int k^2dk\int d\Omega = 4\pi\int k^2dk$$
$$\int d^3kk\hat{k}_x^2 = \int k^2dk\int \hat{k}_x^2d\Omega = \int k^2dk\int\sin^3\theta d\theta\int\cos\phi d\phi = \frac{4\pi}{3}\int k^2dk$$

after minimal algebra, to further simplify the Eq.(D.39) to

$$\left\langle 0\left|E_{y*}B_{z*}\right|0\right\rangle = \frac{16\pi}{3}\int k^2dkH_{zp}^2(\omega)\gamma_\tau^2\beta_\tau. \tag{D.40}$$

Upon changing the variable of integration from k to ω and substituting the value of the spectral function, we can finally obtain

$$\left\langle 0\left|E_{y*}B_{z*}\right|0\right\rangle = \frac{8\pi}{3}\sinh\frac{2a\tau}{c}\int\frac{\hbar\omega^3}{4\pi^2c^3}d\omega. \tag{D.41}$$

where the relation

$$\gamma_\tau^2\beta_\tau = \cosh^2\frac{a\tau}{c}\tanh\frac{a\tau}{c} = \sinh\frac{a\tau}{c}\cosh\frac{a\tau}{c} = \frac{1}{2}\sinh\frac{2a\tau}{c} \tag{D.42}$$

was used.

D.2.7 $\langle 0\,|E_{z*}B_{x*}|\,0\rangle$

$$\langle 0\,|E_{z*}B_{x*}|\,0\rangle = \sum_{\lambda=1}^{2}\int d^3kH_{zp}^2(\omega)\gamma_\tau\left[\hat{\epsilon}_z - \beta_\tau\left(\hat{k}\times\hat{\epsilon}\right)_y\right]\left(\hat{k}\times\hat{\epsilon}\right)_x \tag{D.43}$$

with the polarization equations,

$$\sum_{\lambda=1}^{2}\hat{\epsilon}_z\left(\hat{k}\times\hat{\epsilon}\right)_x = \hat{k}_y$$

$$\sum_{\lambda=1}^{2}\left(\hat{k}\times\hat{\epsilon}\right)_y\left(\hat{k}\times\hat{\epsilon}\right)_x = -\hat{k}_x\hat{k}_y$$

reduces to

$$\langle 0\,|E_{z*}B_{x*}|\,0\rangle = \int k^2dkH_{zp}^2(\omega)\gamma_\tau\left[\hat{k}_y + \beta_\tau\hat{k}_x\hat{k}_y\right]. \tag{D.44}$$

These angular integrations have already been found in Eq.(D.27) and Eq.(D.28) to be zero. Therefore, we conclude that

$$\langle 0\,|E_{z*}B_{x*}|\,0\rangle = 0. \tag{D.45}$$

This result is also expected due to symmetry about the $x-$axis, the direction of the objects accelerated motion. Hence the value of $\langle 0\,|E_{z*}B_{x*}|\,0\rangle$ should be the same as that of $\left\langle 0\left|E_{y*}B_{x*}\right|0\right\rangle$, which is zero.

D.2.8 $\left\langle 0\left|E_{z*}B_{y*}\right|0\right\rangle$

Once again due to the symmetry around the $x-$axis, the value of $\left\langle 0\left|E_{z*}B_{y*}\right|0\right\rangle$ should be at least proportional to that of $\left\langle 0 \left| E_{y*} B_{z*} \right| 0 \right\rangle$. $\left\langle 0 \left| E_{z*} B_{y*} \right| 0 \right\rangle$ is expressed as

$$\left\langle 0 \left| E_{z*} B_{y*} \right| 0 \right\rangle = \sum_{\lambda=1}^{2} \int d^3 k H_{zp}^2(\omega)$$
$$\times \gamma_\tau \left[\hat{\epsilon}_z - \beta_\tau \left(\hat{k} \times \hat{\epsilon} \right)_y \right] \gamma_\tau \left[\left(\hat{k} \times \hat{\epsilon} \right)_y - \beta_\tau \hat{\epsilon}_z \right], \qquad \text{(D.46)}$$

which can be simplified with the polarization equations,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z \left(\hat{k} \times \hat{\epsilon} \right)_y = -\hat{k}_x$$
$$\sum_{\lambda=1}^{2} \left(\hat{k} \times \hat{\epsilon} \right)_y \left(\hat{k} \times \hat{\epsilon} \right)_y = \hat{k}_x^2 + \hat{k}_z^2 = 1 - \hat{k}_y^2$$
$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z^2 = 1 - \hat{k}_z^2.$$

The first equation above reduces to zero after the angular integration Eq.(D.38). Substituting the other two results, Eq.(D.46) simplifies to

$$\left\langle 0 \left| E_{z*} B_{y*} \right| 0 \right\rangle = \int d^3 k H_{zp}^2(\omega) \left(-\gamma_\tau^2 \beta_\tau \right) \left[\left(1 - \hat{k}_y^2 \right) + \left(1 - \hat{k}_z^2 \right) \right]$$
$$= \int d^3 k H_{zp}^2(\omega) \left(-\gamma_\tau^2 \beta_\tau \right) \left[1 + \hat{k}_x^2 \right], \qquad \text{(D.47)}$$

where we have used the relation $1 = \hat{k}_x^2 + \hat{k}_y^2 + \hat{k}_z^2$ in the last step. As expected, we obtain the same expression as the Eq(D.39) in $\left\langle 0 \left| E_{y*} B_{z*} \right| 0 \right\rangle$, except the absence of the negative sign. Therefore, we conclude that

$$\left\langle 0 \left| E_{z*} B_{y*} \right| 0 \right\rangle = - \left\langle 0 \left| E_{y*} B_{z*} \right| 0 \right\rangle$$
$$= \frac{8\pi}{3} \sinh \frac{2a\tau}{c} \int \frac{\hbar \omega^3}{4\pi^2 c^3} d\omega. \qquad \text{(D.48)}$$

As mentioned in the main text, only $\left\langle 0 \left| E_{y*} B_{z*} \right| 0 \right\rangle$ and $\left\langle 0 \left| E_{z*} B_{y*} \right| 0 \right\rangle$ remain non-vanishing and the other seven terms reduce to zero.

D.2.9 $\langle 0 | E_{z*} B_{z*} | 0 \rangle$

The value of $\langle 0 | E_{z*} B_{z*} | 0 \rangle$ should vanish as in the case of $\left\langle 0 \middle| E_{y*} B_{y*} \middle| 0 \right\rangle$, for there exists a cylindrical symmetry around the motion of the object, i.e., x−axis.

$$\begin{aligned}
\langle 0 | E_{z*} B_{z*} | 0 \rangle = & \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \\
& \times \gamma_\tau \left[\hat{\epsilon}_z - \beta_\tau \left(\hat{k} \times \hat{\epsilon} \right)_y \right] \gamma_\tau \left[\left(\hat{k} \times \hat{\epsilon} \right)_z + \beta_\tau \hat{\epsilon}_y \right]
\end{aligned} \tag{D.49}$$

Two of the four terms in the equation above can be found immediately as zero after the polarization formula,

$$\begin{aligned}
\sum_{\lambda=1}^{2} \hat{\epsilon}_z \left(\hat{k} \times \hat{\epsilon} \right)_z &= 0 \\
\sum_{\lambda=1}^{2} \left(\hat{k} \times \hat{\epsilon} \right)_y \hat{\epsilon}_y &= 0.
\end{aligned}$$

The remaining two terms turn out to be

$$\sum_{\lambda=1}^{2} \left(\hat{k} \times \hat{\epsilon} \right)_y \left(\hat{k} \times \hat{\epsilon} \right)_z = -\hat{k}_y \hat{k}_z,$$

and

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z \hat{\epsilon}_y = -\hat{k}_y \hat{k}_z.$$

These two terms, however, cancel each other and as expected, we have

$$\begin{aligned}
\langle 0 | E_{z*} B_{z*} | 0 \rangle &= \sum_{\lambda=1}^{2} \int d^3k H_{zp}^2(\omega) \gamma_\tau^2 \beta_\tau \left[-\left(\hat{k} \times \hat{\epsilon} \right)_y \left(\hat{k} \times \hat{\epsilon} \right)_z + \hat{\epsilon}_z \hat{\epsilon}_y \right] \\
&= \int d^3k H_{zp}^2(\omega) \gamma_\tau^2 \beta_\tau \left[\hat{k}_y \hat{k}_z - \hat{k}_y \hat{k}_z \right] \\
&= 0.
\end{aligned} \tag{D.50}$$

This result also confirms the fact that $\mathbf{E} \cdot \mathbf{B} = 0$.

E Derivation of the Momentum Four-Vector of the Electromagnetic Field

In this section, the expressions for the four-momentum (5.15)–(5.17), that is

$$P^\mu = \left(\frac{1}{c}W, \mathbf{P}\right) \tag{E.1}$$

with

$$W = \gamma \int U d\sigma - \frac{\gamma\beta}{c} \int \mathbf{S} \cdot \hat{\mathbf{n}} d\sigma \tag{E.2}$$

and

$$\mathbf{P} = \frac{\gamma}{c^2} \int \mathbf{S} d\sigma + \frac{\gamma\beta}{c} \int \overleftrightarrow{\mathbf{T}} \cdot \hat{\mathbf{n}} d\sigma. \tag{E.3}$$

are going to be derived. For this purpose, we start from the quantity

$$P^\mu \equiv \frac{1}{c} \int \Theta^{\mu\nu} d\sigma_\nu, \tag{E.4}$$

the integration of the electromagnetic energy tensor over a spacelike surface σ given by the equation

$$n^\mu x_\mu + c\tau = 0, \tag{E.5}$$

where n^μ is the unit normal vector of the plane, which is necessarily timelike,

$$n_\mu n^\mu = -1. \tag{E.6}$$

Also by taking the derivative of (E.5), we obtain

$$n_\mu = -c\partial_\mu \tau \quad \text{and} \quad n^\mu = \frac{dx^\mu}{cd\tau}. \tag{E.7}$$

As explained earlier, any instant of an inertial observer is characterized by this space-like plane σ and the unit normal n^ν, and the surface element is given by

$$d\sigma^\mu = n^\mu d\sigma, \tag{E.8}$$

with the invariant area element

$$d\sigma = -n_\mu d\sigma^\mu = dxdydz. \tag{E.9}$$

For the evaluation of the zero-component of the momentum four-vector, we start from the definition (E.1) and obtain

$$P^0 = \frac{1}{c}\int \Theta^{0\nu} d\sigma_\nu = \frac{1}{c}\left[\int \Theta^{00} n_0 d\sigma + \int \Theta^{0k} n_k d\sigma\right], \tag{E.10}$$

where (E.8) was used. In the case of our interest where the object is moving in the positive x-direction with velocity v, the normal surface is given by $n^\nu = (\gamma; \gamma\beta\hat{\mathbf{n}})$, and the above equation becomes

$$P^0 = \frac{1}{c}\left[\int (-U)(-\gamma)d\sigma + \int \left(-\frac{1}{4\pi}\right)(\mathbf{E}\times\mathbf{B})_k\, \gamma\beta\hat{\mathbf{n}}_k d\sigma\right] \tag{E.11}$$

after substituting the energy-momentum tensor elements

$$\Theta^{00} = \frac{1}{8\pi}\left(E^2 + B^2\right) \equiv -U, \tag{E.12}$$

and

$$\Theta^{0i} = -\frac{1}{4\pi}(\mathbf{E}\times\mathbf{B})_i\,, \tag{E.13}$$

where U is the electromagnetic energy density. With the identification of the Poynting vector

$$\mathbf{S} \equiv \frac{c}{4\pi}(\mathbf{E}\times\mathbf{B})\,, \tag{E.14}$$

the quantity inside the square bracket becomes

$$W = \int \gamma U d\sigma - \int \left(\frac{\gamma\beta}{c}\right) \mathbf{S} \cdot \hat{\mathbf{n}} d\sigma, \tag{E.15}$$

which yields our expected results,

$$P^0 \equiv \frac{1}{c} W \tag{E.16}$$

with

$$W = \gamma \int U d\sigma - \frac{\gamma\beta}{c} \int \mathbf{S} \cdot \hat{\mathbf{n}} d\sigma. \tag{E.17}$$

The space part of the momentum four-vector can be derived in a similar manner. We start again from the definition (E.1) and obtain

$$P^i = \frac{1}{c} \int \Theta^{i\nu} d\sigma_\nu = \frac{1}{c} \left[\int \Theta^{i0} n_0 d\sigma + \int \Theta^{ij} n_j d\sigma \right], \tag{E.18}$$

where we have used (E.8). Substituting the space-time mixed elements (E.13) and the space elements of the tensor

$$\Theta_{ij} = T_{ij} = \frac{1}{4\pi} \left[E_i E_j + B_i B_j - \frac{1}{2}(E^2 + B^2)\delta_{ij} \right], \tag{E.19}$$

Eq.(E.18) becomes

$$\begin{aligned} P^i &= \frac{1}{c} \left[-\int \frac{1}{4\pi} (\mathbf{E} \times \mathbf{B})_i (-\gamma) d\sigma + \int T_{ij} \gamma\beta \hat{\mathbf{n}}_j d\sigma \right] \\ &= \frac{1}{c} \left[\frac{\gamma}{c} \int S_i d\sigma + \gamma\beta \int T_i \cdot \hat{\mathbf{n}} d\sigma \right] \\ &= \frac{\gamma}{c^2} \int S_i d\sigma + \frac{\gamma\beta}{c} \int T_i \cdot \hat{\mathbf{n}} d\sigma. \end{aligned} \tag{E.20}$$

In the expression above, T_i is a row vector and $i = x, y, z$. Therefore, in vector notation, we obtain the desired results

$$\mathbf{P} = \frac{\gamma}{c^2} \int \mathbf{S} d\sigma + \frac{\gamma\beta}{c} \int \mathbf{T} \cdot \hat{\mathbf{n}} d\sigma. \tag{E.21}$$

F Derivation of Davies-Unruh Effect

F.1 Overview

Davies-Unruh effect was discovered independently by Davies (1975)[20] and Unruh (1976)[21] in their efforts to better understand the so-called black-hole evaporation in the context of QED. Its remarkable result is summarized as follows: A system undergoing a uniform acceleration a behaves as if it were immersed in a thermal radiation of temperature T that is proportional to the magnitude of the acceleration, namely,

$$T = \frac{\hbar a}{2\pi k_B c}. \tag{F.1}$$

It is hard to understand why an accelerating object sees a thermal radiation of temperature proportional to acceleration, based on the idea of "empty" vacuum. However, once it is realized that the vacuum is filled with ZPF, this Davies-Unruh effect could be understood as a result of the interaction between the accelerating object and the ZPF.

Davies-Unruh effect has been derived in several different ways by Boyer[40, 18, 41], all in the context of SED. In this chapter, Boyer's first method[40] is followed using the two-point correlation function (expectation value), but instead of SED, it is performed in the quantum formulation. [8] First, the correlation function for an object accelerated in ZPF is evaluated, and this value is compared with the value of another correlation function obtained for random thermal radiation of temperature T. By comparison of two expectation values, we can obtain the relationship between the acceleration a of the object and the temperature T of the thermal radiation.

As a basis of this analysis, we adopt a hyperbolic motion[29, 30], in which an object is under constant acceleration.[9] The accelerating object is moving in the positive x-direction, and in its own rest frame S, the object is at rest at a point $(c^2/a, 0, 0)$, which

[8] A similar derivation in the case of a scalar field is also found in Milonni[42], Sec. 2.10.

[9] For more detailed descriptions of the hyperbolic motion, refer to Sec. 4.2.

coincides with the laboratory inertial frame I_* at $\tau = t = 0$. The space and time coordinates of the object in the laboratory frame I_* is related to the proper time τ in the following way:

$$\begin{aligned} t_* &= \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right), \\ x_* &= \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right), \\ y_* &= 0, \\ z_* &= 0, \end{aligned} \tag{F.2}$$

and also,

$$\beta_\tau = \frac{u_*(\tau)}{c} = \frac{1}{c}\frac{dx_*}{dt_*} = \frac{1}{c}\frac{dx_*/d\tau}{dt_*/d\tau} = \tanh\left(\frac{a\tau}{c}\right), \tag{F.3}$$

and

$$\gamma_\tau = \frac{1}{\sqrt{1-\beta_\tau^2}} = \frac{1}{sech(a\tau/c)} = \cosh\left(\frac{a\tau}{c}\right). \tag{F.4}$$

F.2 Massless Scalar Field

F.2.1 ZPF in a Massless Scalar Field

For a massless scalar field, the ZPF can be expressed as an expansion of plane waves with random phases:

$$\overline{\phi}(\mathbf{r}, t) = \int d^3k\ f_q(\omega)\left[\alpha(\mathbf{k})\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r}) + \alpha^\dagger(\mathbf{k})\exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})\right], \tag{F.5}$$

where $f_q(\omega)$ is the spectral function introduced to set the scale of ZPF, and has the value, [10]

$$f_q^2(\omega) = \frac{\hbar c^2}{4\pi^2 \omega}. \tag{F.6}$$

Also, $\alpha(\mathbf{k})$ and $\alpha^\dagger(\mathbf{k})$ are annihilation and creation operators, which follow the commutation rules

$$[\alpha(\mathbf{k}_1),\ \alpha(\mathbf{k}_2)] = \left[\alpha^\dagger(\mathbf{k}_1),\ \alpha^\dagger(\mathbf{k}_2)\right] = 0 \tag{F.7}$$

$$\left[\alpha(\mathbf{k}_1),\ \alpha^\dagger(\mathbf{k}_2)\right] = \delta^3(\mathbf{k}_1 - \mathbf{k}_2) \tag{F.8}$$

and have the expectation values,

$$\langle 0 \left|\alpha(\mathbf{k}_1)\ \alpha(\mathbf{k}_2)\right| 0\rangle = \left\langle 0 \left|\alpha^\dagger(\mathbf{k}_1)\ \alpha^\dagger(\mathbf{k}_2)\right| 0\right\rangle = 0, \tag{F.9}$$

$$\left\langle 0 \left|\alpha(\mathbf{k}_1)\ \alpha^\dagger(\mathbf{k}_2)\right| 0\right\rangle = \delta^3\left(\mathbf{k}_1 - \mathbf{k}_2\right), \tag{F.10}$$

$$\left\langle 0 \left|\alpha^\dagger(\mathbf{k}_1)\ \alpha(\mathbf{k}_2)\right| 0\right\rangle = 0. \tag{F.11}$$

The overline on ϕ in Eq.(F.5) indicates that this field is expressed in operators. Notice the absence of the polarization vectors in the case of scalar field, as compared to the ordinary vector field such as Eq.(2.12) and Eq.(2.13).

F.2.2 Expectation Value for an Accelerating Object in Random Zero-Point Radiation

Now we like to evaluate the expectation values in the field fluctuations at a point $\mathbf{r}$ in space, which characterizes the random radiation field. For this purpose, we construct

[10] In the reference[40], Boyer uses $f_0(\omega) = \hbar c^2/2\pi^2\omega$ for the classical case, and in the quantum case, extra factor of $1/2$ is inserted to the expression for the field, Eq.(F.5) with the same $f_0(\omega)$. However, it seems that an extra factor of $1/\sqrt{2}$ would be more appropriate to attain the correspondence between the classical and the quantum cases. In the present research, this extra factor of $1/\sqrt{2}$ is inserted in the function $f_q(\omega)$, so that the expression of the field remains unchanged in the quantum case except the use of quantum operators and the exponential functions instead of cosine functions. For more details on this difference in the scaling factor between the classical and the quantum cases, refer to Appendix B.

the product of fields at the point $\mathbf{r}$ and at two different times $\sigma - \tau/2$ and $\sigma + \tau/2$, i.e., $\langle 0 \,|\, \overline{\varphi}(0, \sigma - \tau/2)\, \overline{\varphi}(0, \sigma + \tau/2) |\, 0 \rangle$.

Since the ZPF spectrum is Lorentz-invariant, the field $\overline{\varphi}(0, \sigma \pm \tau/2)$ at the location of the object in the inertial frame I_τ in which the object is instantaneously at rest may be equivalent to the field in the laboratory frame I_*, whose space and time coordinates are related to those in I_τ frame by the Lorentz transformations (F.2), (F.3), and (F.4), that is,

$$\overline{\varphi}(0, \sigma \pm \tau/2) = \overline{\phi} \left[\frac{c^2}{a} \cosh\left(\frac{a\,(\sigma \pm \tau/2)}{c} \right), 0, 0, \frac{c}{a} \sinh\left(\frac{a\,(\sigma \pm \tau/2)}{c} \right) \right], \tag{F.12}$$

where $\overline{\phi}$ is the field in the laboratory frame I_*, but the coordinates are given in terms of the object's proper time τ.

It is to be noted that, since the field is expressed by field operators, in evaluating the expectation value, the order of the operators does affect the result, which is not an issue in the case of classical random radiation. Thus, to evaluate the expectation value, we construct a symmetrized product of operators such that

$$\langle 0 \,|\, \overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2) |\, 0 \rangle^{\ddagger} = \frac{1}{2} \langle 0 \,|\{ \overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2)\}|\, 0 \rangle\,, \tag{F.13}$$

where the double dagger on the left hand side of the equation indicates that the product is yet to be symmetrized, and $\{\overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2)\}$ inside the bracket on the right hand side is an anti-commutator, defined as

$$\{\overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2)\} = \overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2) + \overline{\varphi}(\mathbf{r}_2, t_2)\, \overline{\varphi}(\mathbf{r}_1, t_1), \tag{F.14}$$

yielding,

$$\langle 0 \,|\, \overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2) |\, 0 \rangle^{\ddagger} = \frac{1}{2} \left\{ \langle 0 \,| \overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2) |\, 0 \rangle + \langle 0 \,| \overline{\varphi}(\mathbf{r}_2, t_2)\, \overline{\varphi}(\mathbf{r}_1, t_1) |\, 0 \rangle \right\}, \tag{F.15}$$

for the expectation value to be evaluated. Upon substituting the expression for the ZPF from (F.5), we obtain for the first term of the equation above,

$$\langle 0 \,|\, \overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2) |\, 0 \rangle = \int d^3k_1 \int d^3k_2\, f_q(\omega_1)\, f_q(\omega_2) \\ \times \left\langle 0 \left| \left[\alpha(\mathbf{k_1}) e^{i\Theta_1} + \alpha^\dagger(\mathbf{k_1}) e^{-i\Theta_1} \right] \left[\alpha(\mathbf{k_2}) e^{i\Theta_2} + \alpha^\dagger(\mathbf{k_2}) e^{-i\Theta_2} \right] \right| 0 \right\rangle, \tag{F.16}$$

where

$$\Theta_1 = \mathbf{k}_1 \cdot \mathbf{r}_1 - \omega_1 t_1, \tag{F.17}$$

$$\Theta_2 = \mathbf{k}_2 \cdot \mathbf{r}_2 - \omega_2 t_2. \tag{F.18}$$

This equation has four terms, but with the use of the relationship (F.9)- (F.11), three terms are found to vanish and the expression simplifies to

$$\langle 0 \,|\, \overline{\varphi}(\mathbf{r}_1, t_1)\, \overline{\varphi}(\mathbf{r}_2, t_2) |\, 0 \rangle \\ = \int d^3k_1 \int d^3k_2\, f_q(\omega_1)\, f_q(\omega_2) e^{i\Theta_1} e^{-i\Theta_2} \left\langle 0 \left| \alpha(\mathbf{k_1}) \alpha^\dagger(\mathbf{k_2}) \right| 0 \right\rangle \tag{F.19}$$

The expectation value yields a delta function in **k**, as shown in (F.10), which after one integration over **k** reduces the expression above to

$$\int d^3k\, f_q^2(\omega) \exp\left[i\, (\mathbf{k} \cdot \mathbf{r}_1 - \omega t_1) \right] \exp\left[-i\, (\mathbf{k} \cdot \mathbf{r}_2 - \omega t_2) \right] \\ = \int d^3k\, f_q^2(\omega) \exp\left\{ i \left[\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2) - \omega\, (t_1 - t_2) \right] \right\}. \tag{F.20}$$

It is easy to show that, after following the same steps, the second term yields similar result as (F.20):

$$\int d^3k\, f_q^2(\omega) \exp\left[i\left(\mathbf{k}\cdot\mathbf{r}_2 - \omega t_2\right)\right] \exp\left[-i\left(\mathbf{k}\cdot\mathbf{r}_1 - \omega t_1\right)\right]$$
$$= \int d^3k\, f_q^2(\omega) \exp\left\{-i\left[\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2) - \omega\,(t_1 - t_2)\right]\right\}, \quad \text{(F.21)}$$

with the only difference of (F.21) from (F.20) in the sign of the argument of the exponential function. Therefore, after adding the two results above, the exponential functions are replaced by a cosine, and the expectation value (F.15) becomes

$$\begin{aligned}
\langle 0\,|\,\overline{\varphi}(\mathbf{r}_1, t_1)\,\overline{\varphi}(\mathbf{r}_2, t_2)|\,0\rangle^{\ddagger} &= \frac{1}{2}\left\{\langle 0\,|\overline{\varphi}(\mathbf{r}_1, t_1)\,\overline{\varphi}(\mathbf{r}_2, t_2)|\,0\rangle + \langle 0\,|\overline{\varphi}(\mathbf{r}_2, t_2)\,\overline{\varphi}(\mathbf{r}_1, t_1)|\,0\rangle\right\}\\
&= \int d^3k\, f_q^2(\omega) \cos\left[\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2) - \omega\,(t_1 - t_2)\right]\\
&= \int d^3k \frac{\hbar c^2}{4\pi^2\omega} \cos\left[\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}_2) - \omega\,(t_1 - t_2)\right], \qquad \text{(F.22)}
\end{aligned}$$

where the value of $f_q(\omega)$ in (F.6) was inserted in the second equality. After substituting the value of $\mathbf{r}$ and t from (F.12), specifically

$$\mathbf{r}_1 = \frac{c^2}{a}\cosh\left(\frac{a(\sigma - \tau/2)}{c}\right), \; t_1 = \frac{c}{a}\sinh\left(\frac{a(\sigma - \tau/2)}{c}\right) \qquad \text{(F.23)}$$
$$\mathbf{r}_2 = \frac{c^2}{a}\cosh\left(\frac{a(\sigma + \tau/2)}{c}\right), \; t_2 = \frac{c}{a}\sinh\left(\frac{a(\sigma - \tau/2)}{c}\right), \qquad \text{(F.24)}$$

the above expression changes to

$$\begin{aligned}
&\langle 0\,|\,\overline{\varphi}(0, \sigma - \tau/2)\,\overline{\varphi}(0, \sigma + \tau/2)|\,0\rangle^{\ddagger}\\
&\quad = \int d^3k \frac{\hbar c^2}{4\pi^2\omega} \cos\left\{k_x \frac{c^2}{a}\left[\cosh\left(\frac{a(\sigma - \tau/2)}{c}\right) - \cosh\left(\frac{a(\sigma + \tau/2)}{c}\right)\right]\right.\\
&\qquad\qquad \left. -\omega\frac{c}{a}\left[\sinh\left(\frac{a(\sigma - \tau/2)}{c}\right) - \sinh\left(\frac{a(\sigma + \tau/2)}{c}\right)\right]\right\}. \qquad \text{(F.25)}
\end{aligned}$$

This expression includes the proper time σ in the argument of the cosine function. However, since there is no preferred time in the hyperbolic motion, this dependence on

σ should vanish eventually, as will be shown below.

(F.25) can be more easily evaluated with the use of standard Lorentz transformation,

$$\omega' = \omega \cosh(a\sigma/c) - ck_x \sinh(a\sigma/c), \tag{F.26}$$

$$k'_x = k_x \cosh(a\sigma/c) - \omega c \sinh(a\sigma/c), \tag{F.27}$$

$$k'_y = k_y, \quad k'_z = k_z, \tag{F.28}$$

and the Jacobian of the transformation

$$d^3k = d^3k' \gamma(1 + v k'_x/\omega'). \tag{F.29}$$

With these transformations, we find that

$$\frac{d^3k}{\omega} = \frac{d^3k' \gamma(1 + v k'_x/\omega')}{\gamma(\omega' + v k'_x)} = \frac{d^3k'}{\omega'}, \tag{F.30}$$

and the expansion,

$$\cosh(x) = 1 + x^2/2! + \cdots \simeq 1 + x^2/2, \tag{F.31}$$

$$\sinh(x) = x + x^3/3! + \cdots \simeq x, \tag{F.32}$$

together with (F.30) simplifies (F.25) as

$$\langle 0 \,|\, \overline{\varphi}(0, \sigma - \tau/2)\, \overline{\varphi}(0, \sigma + \tau/2) |\, 0 \rangle^{\ddagger} = \int \frac{\hbar c^2}{4\pi^2} \frac{d^3k'}{\omega'} \cos\left[2\omega' \frac{c}{a} \sinh\left(a\tau/2c \right) \right]. \tag{F.33}$$

Note that this expression has no σ dependence as expected, for there is no preferred time in hyperbolic motion. We integrate this equation over the angles using

$$\int d^3k = \int k^2 dk \int d\Omega = 4\pi \int k^2 dk, \tag{F.34}$$

and change the variable from k to $\omega = kc$ to obtain

$$\begin{aligned} &\langle 0 \,|\, \overline{\varphi}(0, \sigma - \tau/2)\, \overline{\varphi}(0, \sigma + \tau/2) |\, 0\rangle^{\ddagger} \\ &\qquad = \frac{\hbar}{\pi c} \int d\omega' \omega' \cos\left[2\omega' \frac{c}{a} \sinh\left(a\tau/2c\right)\right]. \end{aligned} \tag{F.35}$$

This function is of the form

$$\begin{aligned} \int_0^\infty dx\ x\ \cos bx &= \operatorname{Re} \lim_{\lambda -> 0} \int_0^\infty dx\ x\ \exp[(ib - \lambda)x] \\ &= -b^{-2}, \end{aligned} \tag{F.36}$$

and we can find the expectation value (F.33) to be

$$\langle 0 \,|\, \overline{\varphi}(0, \sigma - \tau/2)\, \overline{\varphi}(0, \sigma + \tau/2) |\, 0\rangle^{\ddagger} = -\frac{\hbar a^2}{4\pi c^3} \operatorname{csch}^2\left(\frac{a\tau}{2c}\right). \tag{F.37}$$

F.2.3 Expectation Value for an Accelerating Object in Random Thermal Radiation

We now explore the expectation value for a point detector *at rest* in a thermal radiation field and compare this value with (F.37). It will be found that the two expectation values agree if $T = \hbar a/2\pi k_B c$.

The object is at rest in its own inertial frame, and this time the surrounding field is a random thermal radiation of temperature T on top of the zero-point field. Therefore, the object will see both the ZPF and the thermal radiation so that

$$f_{qT}^2(\omega) = \frac{\hbar c^2}{2\omega}\left(\frac{1}{2} + \frac{1}{\exp(\hbar\omega/kT) - 1}\right) = \frac{\hbar c^2}{4\omega} \coth\left(\frac{\hbar\omega}{2kT}\right). \tag{F.38}$$

The evaluation of the expectation value follows similar steps as the ones in the previous section: the product of the fields at two different times $s \pm t/2$, $\left\langle 0 \left| \,\overline{\phi}_T(0, s-t/2)\, \overline{\phi}_T(0, s+t/2) \right| 0 \right\rangle^{\ddagger}$ is constructed, and the expression for the scalar field (F.5) inserted. With the use of the symmetrized operators (F.13), each of the two terms are shown to have similar forms except the sign in the exponential functions, which transforms to a cosine function after the addition. The argument of the hyperbolic sine function in (F.33) is further expanded to obtain a simpler expression,

$$\cos\left[2\omega\frac{c}{a}\sinh\left(at/2c\right)\right] \simeq \cos\left[2\omega\frac{c}{a}\left(\frac{at}{2c}\right)\right] \simeq \cos(\omega t). \tag{F.39}$$

Thus, for the expectation value of the object at rest in a thermal radiation, we obtain

$$\left\langle 0 \left| \,\overline{\phi}_T(0, s-t/2)\, \overline{\phi}_T(0, s+t/2) \right| 0 \right\rangle^{\ddagger} = \frac{\hbar}{\pi c}\int_0^\infty d\omega\, \omega\, \coth\left(\frac{\hbar\omega}{2kT}\right)\cos\omega t. \tag{F.40}$$

We break up the integral into two parts using the identity,

$$\coth\left(\frac{\hbar\omega}{2kT}\right) = 1 + \frac{2}{\exp(\hbar\omega/kT) - 1} \tag{F.41}$$

to obtain

$$\int_0^\infty d\omega\, \omega\, \coth\left(\frac{\hbar\omega}{2kT}\right)\cos\omega t = \int_0^\infty d\omega\, \omega\, \cos\omega t + \int_0^\infty d\omega \frac{2\omega\cos\omega t}{\exp(\hbar\omega/kT) - 1}. \tag{F.42}$$

The first term is of the same form as (F.36), and the second term of the form[43]

$$\int_0^\infty dx \frac{x^{2m+1}\cos bx}{e^x - 1} = (-1)^m \frac{\partial^{2m+1}}{\partial b^{2m+1}}\left(\frac{\pi}{2}\coth\pi b - \frac{1}{2b}\right), \quad b > 0. \tag{F.43}$$

Combining these two results, we obtain

$$\int_0^\infty d\omega\, \omega\, \coth\left(\frac{\hbar\omega}{2kT}\right)\cos\omega t = -\frac{1}{t^2} + \left[\frac{1}{t^2} - \left(\frac{\pi kT}{\hbar}\right)^2 \mathrm{csch}^2\left(\frac{\pi kT t}{\hbar}\right)\right], \tag{F.44}$$

which gives us the expectation value

$$\left\langle 0 \left| \overline{\phi}_T(0, s - t/2)\, \overline{\phi}_T(0, s + t/2) \right| 0 \right\rangle^{\ddagger} = -\frac{\pi k^2 T^2}{\hbar c} \mathrm{csch}^2 \left(\frac{\pi k T t}{\hbar} \right). \tag{F.45}$$

F.2.4 Comparison of Two Expectation Values

We now compare the two expectation values (F.37) and (F.45). (F.37) is the expectation value for an object moving with acceleration a in the ZPF, whereas (F.45) is the expectation value for a *stationary* object in a thermal radiation of temperature T. These two objects under completely different situation have strikingly similar results: their functional forms are the same, and moreover, the two results agree with each other provided that the temperature T and the acceleration a are related to each other in the following way

$$T = \frac{\hbar a}{2\pi k_B c}. \tag{F.46}$$

Thus, this result is understood to indicate that an observer accelerating in a vacuum finds himself immersed in a thermal bath of radiation with temperature T, related to the acceleration a by the relation above.

F.3 Massless Vector Field

F.3.1 ZPF in a Massless Vector Field

We now proceed to investigate the case of an electromagnetic vector field. The ZPF in the electromagnetic vector form with a Lorentz-invariant spectrum is given in (2.12) and (2.13) as

$$\begin{aligned}\overline{\mathbf{E}}(\mathbf{r}, t) = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}\,(\mathbf{k}, \lambda)\, H_{zp}(\omega) \\ \times \left\{ \alpha\,(\mathbf{k}, \lambda) \exp\left[i(\mathbf{k} \cdot \mathbf{r} - \omega t)\right] + \alpha^{\dagger}\,(\mathbf{k}, \lambda) \exp\left[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)\right] \right\},\end{aligned} \tag{F.47}$$

and

$$\overline{\mathbf{B}}(\mathbf{r},t) = \sum_{\lambda=1}^{2}\int d^3k\left(\hat{k}\times\hat{\epsilon}\right)H_{zp}(\omega)$$
$$\times\left\{\alpha\left(\mathbf{k},\lambda\right)\exp\left[i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right]+\alpha^{\dagger}\left(\mathbf{k},\lambda\right)\exp\left[-i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right]\right\}. \quad \text{(F.48)}$$

The overlines on **E** and **B** imply that these fields are expressed as operators. The polarization unit vectors $\hat{\epsilon}\,(\mathbf{k},\lambda)\,(\lambda = 1,2)$ and the wave vector $\mathbf{k}$ are mutually orthogonal, and the function $H_{zp}(\omega)$ is determined so that it corresponds to the electromagnetic energy per normal mode at frequency ω,

$$H_{zp}^2(\omega) = \frac{\hbar\omega}{4\pi^2}. \quad \text{(F.49)}$$

The fields observed by an object under hyperbolic motion in its own instantaneous rest frame I_τ and the fields in the laboratory frame I_* are related to each other by a Lorentz-transformation ([32]). This standard transformation applied to the field expressions (F.47) and (F.48) above gives us the fields $\overline{E}(0,\sigma\pm\tau/2)$ and $\overline{B}(0,\sigma\pm\tau/2)$, experienced by an object under constant acceleration, as seen in the inertial laboratory frame I_* as

$$\overline{\mathbf{E}}(\mathbf{r},t) = \sum_{\lambda=1}^{2}\int d^3k\left\{\hat{x}\hat{\epsilon}_x+\hat{y}\gamma_\tau[\hat{\epsilon}_y-\beta_\tau(\hat{k}\times\hat{\epsilon})_z]+\hat{z}\gamma_\tau[\hat{\epsilon}_z+\beta_\tau(\hat{k}\times\hat{\epsilon})_y]\right\}H_{zp}(\omega)$$
$$\times\left\{\alpha\left(\mathbf{k},\lambda\right)\exp\left[i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right]+\alpha^{\dagger}\left(\mathbf{k},\lambda\right)\exp\left[-i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right]\right\}, \quad \text{(F.50)}$$
$$\overline{\mathbf{B}}(\mathbf{r},t) = \sum_{\lambda=1}^{2}\int d^3k\left\{\hat{x}(\hat{k}\times\hat{\epsilon})_x+\hat{y}\gamma_\tau[(\hat{k}\times\hat{\epsilon})_y+\beta_\tau\hat{\epsilon}_z]+\hat{z}\gamma_\tau[(\hat{k}\times\hat{\epsilon})_z-\beta_\tau\hat{\epsilon}_y]\right\}$$
$$\times H_{zp}(\omega)\left\{\alpha\left(\mathbf{k},\lambda\right)\exp\left[i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right]+\alpha^{\dagger}\left(\mathbf{k},\lambda\right)\exp\left[-i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right]\right\}, \quad \text{(F.51)}$$

where t, x, γ_τ, and β_τ are related to the proper time τ under hyperbolic motion as in (F.2), (F.3), and (F.4).

F.3.2 Expectation Value for an Accelerating Object in Random Zero-Point Radiation

The evaluation of the expectation values follows a similar pattern as the scalar field case: we construct a symmetrized operator involving an anti-commutator as

$$\begin{aligned}\left\langle 0\left|\,\overline{\mathbf{E}}_x(\mathbf{r}_1,t_1)\,\overline{\mathbf{E}}_x(\mathbf{r}_2,t_2)\right|0\right\rangle^{\ddagger} &= \frac{1}{2}\left\langle 0\left|\left\{\overline{\mathbf{E}}_x(\mathbf{r}_1,t_1),\;\overline{\mathbf{E}}_x(\mathbf{r}_2,t_2)\right\}\right|0\right\rangle \\ &= \frac{1}{2}\left[\left\langle 0\left|\overline{\mathbf{E}}_x(\mathbf{r}_1,t_1)\,\overline{\mathbf{E}}_x(\mathbf{r}_2,t_2)+\overline{\mathbf{E}}_x(\mathbf{r}_2,t_2)\,\overline{\mathbf{E}}_x(\mathbf{r}_1,t_1)\right|0\right\rangle\right] \\ &= \frac{1}{2}\left[\left\langle 0\left|\overline{\mathbf{E}}_x(\mathbf{r}_1,t_1)\,\overline{\mathbf{E}}_x(\mathbf{r}_2,t_2)\right|0\right\rangle+\left\langle 0\left|\overline{\mathbf{E}}_x(\mathbf{r}_2,t_2)\,\overline{\mathbf{E}}_x(\mathbf{r}_1,t_1)\right|0\right\rangle\right],\end{aligned} \tag{F.52}$$

and calculate each term separately using the field expressions $\alpha\left(\mathbf{k}_1,\lambda\right)$ (F.50) and (F.51). Thus, the first term in (F.52) becomes

$$\begin{aligned}\left\langle 0\left|\,\overline{\mathbf{E}}_x(\mathbf{r}_1,t_1)\,\overline{\mathbf{E}}_x(\mathbf{r}_2,t_2)\right|0\right\rangle &= \sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3k_1\int d^3k_2\;\hat{\epsilon}_x^2 H_{zp}(\omega_1)H_{zp}(\omega_2) \\ \times\left\langle 0\left|\left\{\alpha\left(\mathbf{k}_1,\lambda\right)e^{i\Theta_1}+\alpha^{\dagger}\left(\mathbf{k}_1,\lambda\right)e^{-i\Theta_1}\right\}\right.\right. &\left.\left.\times\left\{\alpha\left(\mathbf{k}_2,\lambda\right)e^{i\Theta_2}+\alpha^{\dagger}\left(\mathbf{k}_2,\lambda\right)e^{-i\Theta_2}\right\}\right|0\right\rangle\end{aligned} \tag{F.53}$$

where

$$\Theta_1 = \mathbf{k}_1\cdot\mathbf{r}_1-\omega_1 t_1, \tag{F.54}$$

$$\Theta_2 = \mathbf{k}_2\cdot\mathbf{r}_2-\omega_2 t_2, \tag{F.55}$$

and again only one term in the bracket remains unvanishing, while the other three terms reduce to zero, due to the relation (2.16)-(2.18), leaving

$$\begin{aligned}\left\langle 0\left|\,\overline{\mathbf{E}}_x(\mathbf{r}_1,t_1)\,\overline{\mathbf{E}}_x(\mathbf{r}_2,t_2)\right|0\right\rangle &= \sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3k_1\int d^3k_2\;\hat{\epsilon}_x^2 H_{zp}(\omega_1)H_{zp}(\omega_2) \\ &\quad\times e^{i\Theta_1}e^{-i\Theta_2}\left\langle 0\left|\alpha\left(\mathbf{k}_1,\lambda\right)\alpha^{\dagger}\left(\mathbf{k}_2,\lambda\right)\right|0\right\rangle.\end{aligned} \tag{F.56}$$

After integrating over a delta function in **k** that comes out of the bracket as shown in (2.17), we obtain

$$\begin{aligned}\left\langle 0 \left| \overline{\mathbf{E}}_x(\mathbf{r}_1, t_1)\, \overline{\mathbf{E}}_x(\mathbf{r}_2, t_2) \right| 0 \right\rangle &= \sum_{\lambda=1}^{2} \int d^3k\, \hat{\epsilon}_x^2 H_{zp}^2(\omega) \exp\{i\,[\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2) - \omega\,(t_1-t_2)]\} \\ &= \sum_{\lambda=1}^{2} \int d^3k\, \hat{\epsilon}_x^2 \frac{\hbar\omega}{4\pi^2} \exp\{i\,[\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2) - \omega\,(t_1-t_2)]\}\,.\end{aligned} \tag{F.57}$$

Following the same procedures, we can show that the second term also yields a similar result:

$$\begin{aligned}&\left\langle 0 \left| \overline{\mathbf{E}}_x(\mathbf{r}_2, t_2)\, \overline{\mathbf{E}}_x(\mathbf{r}_1, t_1) \right| 0 \right\rangle \\ &\qquad = \sum_{\lambda=1}^{2} \int d^3k\, \hat{\epsilon}_x^2 \frac{\hbar\omega}{4\pi^2} \exp\{-i\,[\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2) - \omega\,(t_1-t_2)]\}\,.\end{aligned} \tag{F.58}$$

Thus, just like the case of a scalar field, the two results above added together replaces the exponential functions with a cosine function, yielding the expression for the expectation value (F.52)

$$\begin{aligned}\left\langle 0 \left| \overline{\mathbf{E}}_x(\mathbf{r}_1, t_1)\, \overline{\mathbf{E}}_x(\mathbf{r}_2, t_2) \right| 0 \right\rangle^{\ddagger} &= \frac{1}{2}\left\{\left\langle 0 \left| \overline{\mathbf{E}}(\mathbf{r}_1, t_1)\, \overline{\mathbf{E}}(\mathbf{r}_2, t_2) \right| 0 \right\rangle + \left\langle 0 \left| \overline{\mathbf{E}}(\mathbf{r}_2, t_2)\, \overline{\mathbf{E}}(\mathbf{r}_1, t_1) \right| 0 \right\rangle\right\} \\ &= \sum_{\lambda=1}^{2} \int d^3k\, \hat{\epsilon}_x^2 \frac{\hbar\omega}{4\pi^2} \cos\,[\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2) - \omega\,(t_1-t_2)]\,,\end{aligned} \tag{F.59}$$

which, upon substituting the value of **r** and t from (F.2), becomes

$$\Big\langle 0 \Big| \, \overline{\mathbf{E}}_x(0, \sigma - \tau/2) \, \overline{\mathbf{E}}_x(0, \sigma + \tau/2) \Big| 0 \Big\rangle^{\ddagger}$$
$$= \sum_{\lambda=1}^{2} \int d^3k \, \hat{\epsilon}_x^2 \frac{\hbar\omega}{4\pi^2} \cos\left\{ k_x \frac{c^2}{a} \left[\cosh\left(\frac{a(\sigma - \tau/2)}{c} \right) - \cosh\left(\frac{a(\sigma + \tau/2)}{c} \right) \right] \right.$$
$$\left. -\omega \frac{c}{a} \left[\sinh\left(\frac{a(\sigma - \tau/2)}{c} \right) - \sinh\left(\frac{a(\sigma + \tau/2)}{c} \right) \right] \right\}. \quad \text{(F.60)}$$

The expression is similar to (F.25) in the scalar field case, except we now have an extra factor of a sum over polarization states. This summation can be evaluated with the use of the polarization formula introduced earlier in Appendix A, specifically, (A.6),

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i \hat{\epsilon}_j = \delta_{ij} - \hat{k}_i \hat{k}_j. \quad \text{(F.61)}$$

With $i = j = 1$, the summation becomes $1 - k_x^2/k$. After introducing the same change of variables as in the scalar case, (F.26)-(F.28), using the Lorentz transformation, we find

$$d^3k (1 - k_x^2/k)\omega = d^3k' (1 - k_x'^2/k)\omega'. \quad \text{(F.62)}$$

The argument of the cosine is also simplified with the expansion (F.31) and (F.32) as before, and the expression (F.60) becomes

$$\Big\langle 0 \Big| \, \overline{\mathbf{E}}_x(0, \sigma - \tau/2) \, \overline{\mathbf{E}}_x(0, \sigma + \tau/2) \Big| 0 \Big\rangle^{\ddagger}$$
$$= \int d^3k' \, \frac{\hbar\omega'}{4\pi^2} \frac{\omega'^2 - c^2 k_x'^2}{\omega'^2} \cos\left[\omega' \frac{2c}{a} \sinh\left(\frac{a\tau}{2c} \right) \right]. \quad \text{(F.63)}$$

The integration over the k-sphere can be divided into the k-integration and the solid angle part as

$$\int d^3k = \int k^2 dk \int d\Omega = \int k^2 dk \int \sin\theta \, d\theta \, d\varphi, \quad \text{(F.64)}$$

which gives

$$\int \frac{d^3k}{4\pi^2} \frac{\omega^2 - c^2k_x^2}{\omega} = \int \frac{k^2dk}{4\pi^2} \frac{c^2k^2 - c^2k_x^2}{ck} \sin\theta \, d\theta \, d\varphi$$
$$= \int \frac{k^2dk}{4\pi^2} ck \int\int (1 - \sin^2\theta \cos^2\varphi) \sin\theta \, d\theta \, d\varphi, \quad \text{(F.65)}$$

where the relation $k_x = k \sin\theta \cos\varphi$ was used, and the prime was omitted for simplicity. The integration over the angles can be easily obtained as $4\pi - 4\pi/3 = 8\pi/3$, and the expression (F.63) becomes

$$\left\langle 0 \left| \, \overline{\mathbf{E}}_x(0, \sigma - \tau/2) \, \overline{\mathbf{E}}_x(0, \sigma + \tau/2) \right| 0 \right\rangle^{\ddagger}$$
$$= \frac{2\hbar}{3\pi c^3} \int_0^\infty d\omega \, \omega^3 \cos\left[\omega \frac{2c}{a} \sinh\left(\frac{a\tau}{2c}\right)\right]. \quad \text{(F.66)}$$

This integration is of the form

$$\int_0^\infty dx \, x^3 \cos bx = \Gamma(3+1) \, b^{-(3+1)} = 6/b^4, \quad \text{(F.67)}$$

and we finally obtain

$$\left\langle 0 \left| \, \overline{\mathbf{E}}_x(0, \sigma - \tau/2) \, \overline{\mathbf{E}}_x(0, \sigma + \tau/2) \right| 0 \right\rangle^{\ddagger} = \frac{4\hbar}{\pi c^3} \left(\frac{a}{2c}\right)^4 \operatorname{csch}^4\left(\frac{a\tau}{2c}\right). \quad \text{(F.68)}$$

We continue with the evaluation of other expectation values: the correlations between $\overline{\mathbf{E}}$ and $\overline{\mathbf{E}}$, $\overline{\mathbf{B}}$ and $\overline{\mathbf{B}}$, $\overline{\mathbf{E}}$ and $\overline{\mathbf{B}}$, and $\overline{\mathbf{B}}$ and $\overline{\mathbf{E}}$. Each of the electric and magnetic fields have three components, making a total of $9x4 = 36$ expectation values. However, from the way the expectation value (F.68) was evaluated, we can see that the basic functional form stays the same just like (F.60), no matter which fields and which components are selected: all of them have the integration over the k-sphere, the summation over the polarization states, the spectral function $H_{zp}^2(\omega)$, and the cosine function which is a result of the addition of two terms involving $e^{i\Theta}$ and $e^{-i\Theta}$. Moreover, we can also observe that the argument of the cosine function is the same regardless of the components chosen, and the only part in (F.60) that changes its form depending on the components is the polarization states. Since this polarization states are scalars, the orders of these states can be switched, which gives

$$\left\langle 0 \middle| \overline{\mathbf{E}}_i(0,\sigma-\tau/2)\,\overline{\mathbf{E}}_j(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger} = \left\langle 0 \middle| \overline{\mathbf{E}}_j(0,\sigma-\tau/2)\,\overline{\mathbf{E}}_i(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger} \tag{F.69}$$

$$\left\langle 0 \middle| \overline{\mathbf{B}}_i(0,\sigma-\tau/2)\,\overline{\mathbf{B}}_j(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger} = \left\langle 0 \middle| \overline{\mathbf{B}}_j(0,\sigma-\tau/2)\,\overline{\mathbf{B}}_i(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger} \tag{F.70}$$

$$\left\langle 0 \middle| \overline{\mathbf{E}}_i(0,\sigma-\tau/2)\,\overline{\mathbf{B}}_j(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger} = \left\langle 0 \middle| \overline{\mathbf{B}}_j(0,\sigma-\tau/2)\,\overline{\mathbf{E}}_i(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger},\ i,j=1,2,3 \tag{F.71}$$

With these in mind, let us see how the correlation of x and y components in electric field turns out. We have

$$\begin{aligned}
&\left\langle 0 \middle| \overline{\mathbf{E}}_x(0,\sigma-\tau/2)\,\overline{\mathbf{E}}_y(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger} \\
&= \sum_{\lambda=1}^{2} \int d^3k \frac{\hbar\omega}{4\pi^2} \hat{\epsilon}_x \gamma_\tau \left[\hat{\epsilon}_y - \beta_\tau (\hat{k}\times\hat{\epsilon})_z\right] \cos\left\{ k_x \frac{c^2}{a}\left[\cosh\left(\frac{a(\sigma-\tau/2)}{c}\right) - \cosh\left(\frac{a(\sigma+\tau/2)}{c}\right)\right]\right. \\
&\qquad\qquad \left. -\omega\frac{c}{a}\left[\sinh\left(\frac{a(\sigma-\tau/2)}{c}\right) - \sinh\left(\frac{a(\sigma+\tau/2)}{c}\right)\right]\right\}.
\end{aligned} \tag{F.72}$$

This involves two summations over polarization states,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x (\hat{k}\times\hat{\epsilon})_z = -\hat{k}_y, \tag{F.73}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x \hat{\epsilon}_y = -\hat{k}_x \hat{k}_y, \tag{F.74}$$

both of which vanish as shown in (C.26) and (C.27). Therefore, we conclude

$$\left\langle 0 \,\middle|\, \overline{\mathbf{E}}_x(0,\sigma_-)\ \overline{\mathbf{E}}_y(0,\sigma_+) \middle|\, 0 \right\rangle^{\ddagger} = 0. \tag{F.75}$$

where

$$\sigma_- = \sigma - \tau/2, \quad \sigma_+ = \sigma + \tau/2. \tag{F.76}$$

The evaluation of $\left\langle 0 \middle| \overline{\mathbf{E}}_x(0,\sigma-\tau/2)\overline{\mathbf{E}}_z(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger}$ involves

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x(\hat{k}\times\hat{\epsilon})_y = \hat{k}_z, \ \ \sum_{\lambda=1}^{2} \hat{\epsilon}_x\hat{\epsilon}_z = -\hat{k}_x\hat{k}_z, \tag{F.77}$$

which also vanish as in (C.20) and (C.21). The function $\left\langle 0 \middle| \overline{\mathbf{E}}_y(0,\sigma-\tau/2)\overline{\mathbf{E}}_x(0,\sigma+\tau/2) \middle| 0 \right\rangle^{\ddagger}$ includes summations

$$\sum_{\lambda=1}^{2} (\hat{k}\times\hat{\epsilon})_z\hat{\epsilon}_x = -\hat{k}_y, \tag{F.78}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y\hat{\epsilon}_x = -\hat{k}_x\hat{k}_y, \tag{F.79}$$

which are exactly the same values as those in the $\left\langle 0 \middle| \overline{\mathbf{E}}_x(0,\sigma_-)\overline{\mathbf{E}}_y(0,\sigma_+) \middle| 0 \right\rangle^{\ddagger}$ case. Therefore, we find that

$$\left\langle 0 \,\middle|\, \overline{\mathbf{E}}_x(0,\sigma-\tau/2)\ \overline{\mathbf{E}}_y(0,\sigma+\tau/2) \middle|\, 0 \right\rangle^{\ddagger} = \left\langle 0 \,\middle|\, \overline{\mathbf{E}}_y(0,\sigma-\tau/2)\ \overline{\mathbf{E}}_x(0,\sigma+\tau/2) \middle|\, 0 \right\rangle^{\ddagger} = 0, \tag{F.80}$$

in partial confirmation of (F.69). This is also physically reasonable, since there should exist a symmetry about the direction of acceleration, i.e., x-axis.

The evaluation of $\left\langle 0 \middle| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{E}}_y(0,\sigma_+) \middle| 0 \right\rangle^{\ddagger}$ requires extended calculations. For this expectation value, we have

$$\begin{aligned}
&\left\langle 0 \left| \, \overline{\mathbf{E}}_y(0, \sigma - \tau/2) \, \overline{\mathbf{E}}_y(0, \sigma + \tau/2) \right| 0 \right\rangle^{\ddagger} \\
&\quad = \sum_{\lambda=1}^{2} \int d^3k \frac{\hbar\omega}{4\pi^2} \left[\hat{\epsilon}_y \cosh\left(\frac{a(\sigma - \tau/2)}{c} \right) - (\hat{k} \times \hat{\epsilon})_z \sinh\left(\frac{a(\sigma - \tau/2)}{c} \right) \right] \\
&\qquad\qquad \times \left[\hat{\epsilon}_y \cosh\left(\frac{a(\sigma + \tau/2)}{c} \right) - (\hat{k} \times \hat{\epsilon})_z \sinh\left(\frac{a(\sigma + \tau/2)}{c} \right) \right] \\
&\qquad\qquad \times \cos\left\{ k_x \frac{c^2}{a} \left[\cosh\left(\frac{a(\sigma - \tau/2)}{c} \right) - \cosh\left(\frac{a(\sigma + \tau/2)}{c} \right) \right] \right. \\
&\qquad\qquad\qquad \left. - \omega \frac{c}{a} \left[\sinh\left(\frac{a(\sigma - \tau/2)}{c} \right) - \sinh\left(\frac{a(\sigma + \tau/2)}{c} \right) \right] \right\}. \qquad \text{(F.81)}
\end{aligned}$$

The summations involved are of the following three types,

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y (\hat{k} \times \hat{\epsilon})_z = \hat{k}_x \qquad \text{(F.82)}$$

$$\sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_z (\hat{k} \times \hat{\epsilon})_z = 1 - \hat{k}_z^2 = \hat{k}_x^2 + \hat{k}_y^2 \qquad \text{(F.83)}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y^2 = 1 - \hat{k}_y^2 = \hat{k}_x^2 + \hat{k}_z^2 \qquad \text{(F.84)}$$

$$. \qquad \text{(F.85)}$$

Introducing the same change of variables (F.26)-(F.28), we find that the first summation (F.82) can be easily shown to vanish after the angular integration,

$$\int d^3k k_x = \int k^2 dk \int k_x d\Omega = \int k^2 dk \int \sin^2\theta \cos\varphi \, d\theta \, d\varphi = 0. \qquad \text{(F.86)}$$

The expectation value (F.81) now becomes

$$\left\langle 0 \right| \overline{\mathbf{E}}_y(0, \sigma - \tau/2)\, \overline{\mathbf{E}}_y(0, \sigma + \tau/2) \left| 0 \right\rangle^{\ddagger}$$
$$= \int d^3k' \frac{\hbar\omega'}{4\pi^2} \left[\cosh^2\left(\frac{a\tau}{2c}\right)\left(1 - \hat{k}_y'^2\right) - \sinh^2\left(\frac{a\tau}{2c}\right)\left(1 - \hat{k}_z'^2\right)\right] \cos\left[\omega' \frac{2c}{a} \sinh\left(\frac{a\tau}{2c}\right)\right]. \tag{F.87}$$

The expression above has two terms, but the comparison of this with previous results shows that each term has the same functional form as (F.63), except that each term in (F.87) has a different component of k, and an extra factor of hyperbolic functions, which does not affect the k-sphere integration. Therefore, the evaluation follows the same pattern (F.63)-(F.65), and the angular integration yields

$$\int \left(1 - \hat{k}_y'^2\right) \sin\theta d\Omega = \int\int (1 - \sin^2\theta \sin^2\varphi) \sin\theta\, d\theta\, d\varphi = 4\pi - 4\pi/3 = 8\pi/3, \tag{F.88}$$

and

$$\int \left(1 - \hat{k}_z'^2\right) \sin\theta d\Omega = \int\int (1 - \cos^2\theta) \sin\theta\, d\theta\, d\varphi = 4\pi - 4\pi/3 = 8\pi/3, \tag{F.89}$$

respectively, the same result for each case $\int(1 - \hat{k}_i^2)d\Omega,\ i = 1, 2, 3$. This simplifies (F.87) to

$$\left\langle 0 \right| \overline{\mathbf{E}}_y(0, \sigma - \tau/2)\, \overline{\mathbf{E}}_y(0, \sigma + \tau/2) \left| 0 \right\rangle^{\ddagger}$$
$$= \frac{2\hbar}{3\pi c^3} \int_0^{\infty} d\omega\, \omega^3 \cos\left[\omega \frac{2c}{a} \sinh\left(\frac{a\tau}{2c}\right)\right] \left[\cosh^2\left(\frac{a\tau}{2c}\right) - \sinh^2\left(\frac{a\tau}{2c}\right)\right], \tag{F.90}$$

which, with the use of the identity $\cosh^2 x - \sinh^2 x = 1$, becomes exactly the same as (F.66). Therefore, we obtain

$$\left\langle 0 \right| \overline{\mathbf{E}}_y(0, \sigma - \tau/2)\, \overline{\mathbf{E}}_y(0, \sigma + \tau/2) \left| 0 \right\rangle^{\ddagger} = \frac{4\hbar}{\pi c^3} \left(\frac{a}{2c}\right)^4 \operatorname{csch}^4\left(\frac{a\tau}{2c}\right). \tag{F.91}$$

The evaluation of $\left\langle 0 \left| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{E}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ involves the summations

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y \hat{\epsilon}_z = -\hat{k}_y \hat{k}_z, \tag{F.92}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i (\hat{k} \times \hat{\epsilon})_i = 0. \tag{F.93}$$

The angular integration of the first summation is

$$\int \hat{k}_y \hat{k}_z \sin\theta \, d\Omega = \int \int (\sin\theta \sin\varphi)(\cos\theta) \sin\theta \, d\theta \, d\varphi = 0, \tag{F.94}$$

proving that $\left\langle 0 \left| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{E}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ is zero. The calculation of $\left\langle 0 \left| \overline{\mathbf{E}}_z(0,\sigma_-)\overline{\mathbf{E}}_x(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ involves the same polarization summation as the case of $\left\langle 0 \left| \overline{\mathbf{E}}_x(0,\sigma_-)\overline{\mathbf{E}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$, which was shown earlier to vanish, again in partial confirmation of (F.69). The value of $\left\langle 0 \left| \overline{\mathbf{E}}_z(0,\sigma_-)\overline{\mathbf{E}}_y(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ should also vanish just like $\left\langle 0 \left| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{E}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ due to symmetry around the x-axis. The function $\left\langle 0 \left| \overline{\mathbf{E}}_z(0,\sigma_-)\overline{\mathbf{E}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ involves the following polarization summations:

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z (\hat{k} \times \hat{\epsilon})_y = -\hat{k}_x \tag{F.95}$$

$$\sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_y (\hat{k} \times \hat{\epsilon})_y = 1 - \hat{k}_y^2 \tag{F.96}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z^2 = 1 - \hat{k}_z^2. \tag{F.97}$$

These results will produce an expression,

$$\begin{aligned} &\left\langle 0 \left| \, \overline{\mathbf{E}}_z(0,\sigma-\tau/2) \, \overline{\mathbf{E}}_z(0,\sigma+\tau/2) \right| 0 \right\rangle^{\ddagger} \\ &= \int d^3k' \frac{\hbar\omega'}{4\pi^2} \left[\cosh^2\left(\frac{a\tau}{2c}\right)\left(1-\hat{k}_z'^2\right) - \sinh^2\left(\frac{a\tau}{2c}\right)\left(1-\hat{k}_y'^2\right) \right] \cos\left[\omega' \frac{2c}{a} \sinh\left(\frac{a\tau}{2c}\right)\right], \end{aligned} \tag{F.98}$$

which is very similar to (F.87). The only difference between the expression above and (F.87) is the switched positions of $\hat{k}_y'^2$ and $\hat{k}_z'^2$. However, as we have seen before in (F.88) and (F.89), since the values of integrations are the same, we can conclude that the expectation value $\left\langle 0 \left| \overline{\mathbf{E}}_z(0,\sigma_-)\overline{\mathbf{E}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ is exactly equal to $\left\langle 0 \left| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{E}}_y(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$.

The calculations for the expectation values $\left\langle 0 \left| \overline{\mathbf{B}}_i(0,\sigma_-)\overline{\mathbf{B}}_j(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ also follow a very similar pattern as those for $\left\langle 0 \left| \overline{\mathbf{E}}_i(0,\sigma_-)\overline{\mathbf{E}}_j(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$. By inspecting the form of the zero-point electric and magnetic fields (F.50) and (F.51), we find that they are interchangeable to each other with the transformation,

$$\hat{\epsilon}_i \Leftrightarrow (\hat{k} \times \hat{\epsilon})_i, \quad v \Leftrightarrow -v. \tag{F.99}$$

Moreover, from the polarization formulae, we have

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i \hat{\epsilon}_j = \sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_i (\hat{k} \times \hat{\epsilon})_j = 1 - \hat{k}_i \hat{k}_j \tag{F.100}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_i (\hat{k} \times \hat{\epsilon})_j = -\sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_i \hat{\epsilon}_j = \varepsilon_{ijk} \hat{k}_k, \tag{F.101}$$

which guarantees a complete correspondence between $\left\langle 0 \left| \overline{\mathbf{E}}_i(0,\sigma_-)\overline{\mathbf{E}}_j(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ and $\left\langle 0 \left| \overline{\mathbf{B}}_i(0,\sigma_-)\overline{\mathbf{B}}_j(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$, when the terms $\sum_{\lambda=1}^{2} \hat{\epsilon}_i \hat{\epsilon}_j$ and $\sum_{\lambda=1}^{2} (\hat{k} \times \hat{\epsilon})_i (\hat{k} \times \hat{\epsilon})_j$ are involved. When the cross terms as (F.101) are involved, the sign would be opposite, but this case is always accompanied by a factor of v, which also changes the sign as in (F.99) with the result of cancelling any sign discrepancies that may exist. Therefore, we can conclude that

$$\left\langle 0 \left| \overline{\mathbf{B}}_i(0,\sigma - \tau/2)\overline{\mathbf{B}}_j(0,\sigma + \tau/2) \right| 0 \right\rangle^{\ddagger} = \left\langle 0 \left| \overline{\mathbf{E}}_i(0,\sigma - \tau/2)\overline{\mathbf{E}}_j(0,\sigma + \tau/2) \right| 0 \right\rangle^{\ddagger}. \tag{F.102}$$

The cases of $\left\langle 0 \left| \overline{\mathbf{E}}_i(0,\sigma_-)\overline{\mathbf{B}}_j(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ has been investigated closely in Appendices C and D. Since the analysis goes in parallel here again, the only term which was non-zero in the previous Appendices will be calculated in detail here. The other terms can be shown easily to vanish here as well. For example, the calculation of $\left\langle 0 \left| \overline{\mathbf{E}}_x(0,\sigma_-)\overline{\mathbf{B}}_x(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ includes the summation $\sum_{\lambda=1}^{2} \hat{\epsilon}_x(\hat{k}\times\hat{\epsilon})_x$, which vanishes after the angular integration. The function $\left\langle 0 \left| \overline{\mathbf{E}}_x(0,\sigma_-)\overline{\mathbf{B}}_y(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ involves summations

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x(\hat{k}\times\hat{\epsilon})_z = -\hat{k}_x\hat{k}_z, \tag{F.103}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x\hat{\epsilon}_y = -\hat{k}_z, \tag{F.104}$$

which also vanish after integration. The case of $\left\langle 0 \left| \overline{\mathbf{E}}_x(0,\sigma_-)\overline{\mathbf{B}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ carries summations

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x(\hat{k}\times\hat{\epsilon})_z = -\hat{k}_y, \tag{F.105}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_x\hat{\epsilon}_y = -\hat{k}_x\hat{k}_y, \tag{F.106}$$

which again vanishes. The diagonal term $\left\langle 0 \left| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{B}}_y(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$ involves the following summations:

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y(\hat{k}\times\hat{\epsilon})_y = 0, \tag{F.107}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z(\hat{k}\times\hat{\epsilon})_z = 0, \tag{F.108}$$

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_y\hat{\epsilon}_z = -\hat{k}_y\hat{k}_z, \tag{F.109}$$

$$\sum_{\lambda=1}^{2} (\hat{k}\times\hat{\epsilon})_y(\hat{k}\times\hat{\epsilon})_z = -\hat{k}_y\hat{k}_z. \tag{F.110}$$

The last two integrations have been shown to vanish in (F.94) after the integration over the solid angle.

For the case of $\left\langle 0 \left| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{B}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$, after constructing the symmetrized operators and adding up the contributions from both terms, we obtain

$$\begin{aligned}
&\left\langle 0 \left| \, \overline{\mathbf{E}}_y(0,\sigma-\tau/2)\, \overline{\mathbf{B}}_z(0,\sigma+\tau/2) \right| 0 \right\rangle^{\ddagger} \\
&\quad = \sum_{\lambda=1}^{2} \int d^3k \frac{\hbar\omega}{4\pi^2} \left[\hat{\epsilon}_y \cosh\left(\frac{a(\sigma-\tau/2)}{c}\right) - (\hat{k}\times\hat{\epsilon})_z \sinh\left(\frac{a(\sigma-\tau/2)}{c}\right)\right] \\
&\qquad \times \left[(\hat{k}\times\hat{\epsilon})_z \cosh\left(\frac{a(\sigma+\tau/2)}{c}\right) - \hat{\epsilon}_y \sinh\left(\frac{a(\sigma+\tau/2)}{c}\right)\right] \\
&\qquad \times \cos\left\{ k_x \frac{c^2}{a} \left[\cosh\left(\frac{a(\sigma-\tau/2)}{c}\right) - \cosh\left(\frac{a(\sigma+\tau/2)}{c}\right)\right]\right. \\
&\qquad\qquad \left. -\omega\frac{c}{a}\left[\sinh\left(\frac{a(\sigma-\tau/2)}{c}\right) - \sinh\left(\frac{a(\sigma+\tau/2)}{c}\right)\right]\right\}. \quad \text{(F.111)}
\end{aligned}$$

The summations involved are exactly the same ones (F.82)-(F.84) as the case of $\left\langle 0 \left| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{E}}_y(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$. After introducing the same change of variables (F.26)-(F.28), the expression (F.111) above becomes

$$\begin{aligned}
&\left\langle 0 \left| \, \overline{\mathbf{E}}_y(0,\sigma-\tau/2)\, \overline{\mathbf{B}}_z(0,\sigma+\tau/2) \right| 0 \right\rangle^{\ddagger} \\
&\quad = \int d^3k' \frac{\hbar\omega'}{4\pi^2} \left\{ \hat{k}'_y + \left[\left(1-\hat{k}'^2_y\right) - \left(1-\hat{k}'^2_z\right)\right]\right\} \cos\left[\omega' \frac{2c}{a}\sinh\left(\frac{a\tau}{2c}\right)\right]. \quad \text{(F.112)}
\end{aligned}$$

The first integration has been shown in (F.86) to vanish. For the second integration, we found previously in (F.88) and (F.89) that both of them yield exactly the same value. Therefore, we conclude that $\left\langle 0 \left| \overline{\mathbf{E}}_y(0,\sigma_-)\overline{\mathbf{B}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger} = 0$.

Finally, the case of $\left\langle 0 \left| \overline{\mathbf{E}}_z(0,\sigma_-)\overline{\mathbf{B}}_z(0,\sigma_+) \right| 0 \right\rangle^{\ddagger}$. involves the following four summations:

$$\sum_{\lambda=1}^{2} \hat{\epsilon}_z(\hat{k}\times\hat{\epsilon})_z = \sum_{\lambda=1}^{2} \hat{\epsilon}_y(\hat{k}\times\hat{\epsilon})_y = 0, \quad \text{(F.113)}$$

$$\sum_{\lambda=1}^{2} (\hat{k}\times\hat{\epsilon})_y(\hat{k}\times\hat{\epsilon})_z = \sum_{\lambda=1}^{2} \hat{\epsilon}_y\hat{\epsilon}_z = -\hat{k}_y\hat{k}_z. \quad \text{(F.114)}$$

The last integration also vanish as already shown in (F.94), which proves that $\left\langle 0 \left| \overline{\mathbf{E}}_z(0, \sigma_-)\overline{\mathbf{B}}_z(0, \sigma_+) \right| 0 \right\rangle^{\ddagger}$ also vanishes. The other terms that were not treated here can also be shown to vanish from the symmetry considerations. For example, the value of $\left\langle 0 \left| \overline{\mathbf{E}}_y(0, \sigma_-)\overline{\mathbf{B}}_x(0, \sigma_+) \right| 0 \right\rangle^{\ddagger}$ has the same value (which is zero) as $\left\langle 0 \left| \overline{\mathbf{E}}_x(0, \sigma_-)\overline{\mathbf{B}}_y(0, \sigma_+) \right| 0 \right\rangle^{\ddagger}$, in partial confirmation of the relation (F.71).

Summarizing the results obtained in this section, we find that

$$\begin{aligned} &\left\langle 0 \left| \overline{\mathbf{E}}_i(0, \sigma - \tau/2)\overline{\mathbf{E}}_j(0, \sigma + \tau/2) \right| 0 \right\rangle^{\ddagger} \\ &\qquad = \left\langle 0 \left| \overline{\mathbf{B}}_i(0, \sigma - \tau/2)\overline{\mathbf{B}}_j(0, \sigma + \tau/2) \right| 0 \right\rangle^{\ddagger} \\ &\qquad = \frac{4\hbar}{\pi c^3} \left(\frac{a}{2c} \right)^4 \operatorname{csch}^4 \left(\frac{a\tau}{2c} \right) \delta_{ij} \end{aligned} \tag{F.115}$$

and

$$\left\langle 0 \left| \overline{\mathbf{E}}_x(0, \sigma_-)\overline{\mathbf{B}}_y(0, \sigma_+) \right| 0 \right\rangle^{\ddagger} = 0, \quad i, j = 1, 2, 3. \tag{F.116}$$

F.3.3 Expectation Value for an Accelerating Object in Random Thermal Radiation

We now study the case of a detector at rest in its own inertial frame in random thermal radiation. The spectral function now has two terms: ZPF spectrum $H_{zp}(\omega)$ and the Planck spectrum, i.e.,

$$h_{qT}^2(\omega) = \frac{\hbar\omega}{2} \left(\frac{1}{2} + \frac{1}{\exp(\hbar\omega/kT) - 1} \right) = \frac{\hbar\omega}{4} \coth \left(\frac{\hbar\omega}{2kT} \right). \tag{F.117}$$

The evaluations of the expectation value proceed in analogous manners as those in the previous sections. We construct symmetrized operators, which yield two terms of different operator orders. Components of the electric fields from (F.50) are substituted, and the spectral function (F.117) inserted. When these two terms are added, a cosine function is obtained. For example, the expectation values of x-components of electric fields at two different times $s \pm t/2$ can be calculated as

$$\left\langle 0 \left| \, \overline{\mathbf{E}}_{Tx}(0, s-t/2) \, \overline{\mathbf{E}}_{Tx}(0, s+t/2) \right| 0 \right\rangle^{\ddagger}$$
$$= \sum_{\lambda=1}^{2} \int d^3k \, \hat{\epsilon}_x^2 \frac{\hbar\omega}{4\pi^2} \coth\left(\frac{\hbar\omega}{2kT}\right) \cos\left\{ k_x \frac{c^2}{a} \left[\cosh\left(\frac{a(s-t/2)}{c}\right) - \cosh\left(\frac{a(s+t/2)}{c}\right) \right] \right.$$
$$\left. -\omega \frac{c}{a} \left[\sinh\left(\frac{a(s-t/2)}{c}\right) - \sinh\left(\frac{a(s+t/2)}{c}\right) \right] \right\}. \quad \text{(F.118)}$$

The argument of the cosine function can be simplified using (F.39), and after the summation over polarization states, the above expression becomes

$$\left\langle 0 \left| \, \overline{\mathbf{E}}_{Tx}(0, s-t/2) \, \overline{\mathbf{E}}_{Tx}(0, s+t/2) \right| 0 \right\rangle^{\ddagger}$$
$$= \int d^3k \, \left(1 - \hat{k}_x^2\right) \frac{\hbar\omega}{4\pi^2} \coth\left(\frac{\hbar\omega}{2kT}\right) \cos\omega t. \quad \text{(F.119)}$$

At this point, it is clear that the polarization summation part and the following angular integrations are unchanged from the calculations in the previous section. Therefore, the terms that vanish in the last section also vanish here as well. We only need to evaluate the diagonal terms for which $i = j$.

After a change of variable, the expression (F.119) becomes

$$\left\langle 0 \left| \, \overline{\mathbf{E}}_{Tx}(0, s-t/2) \, \overline{\mathbf{E}}_{Tx}(0, s+t/2) \right| 0 \right\rangle^{\ddagger}$$
$$= \delta_{ij} \frac{2\hbar}{3\pi c^2} \int_0^\infty d\omega \, \omega^3 \coth\left(\frac{\hbar\omega}{2kT}\right) \cos\omega t, \quad \text{(F.120)}$$

which can be evaluated by breaking up the integral to two parts as

$$\int_0^\infty d\omega \, \omega^3 \coth\left(\frac{\hbar\omega}{2kT}\right) \cos\omega t = \int_0^\infty d\omega \, \omega^3 \cos\omega t + \int_0^\infty d\omega \, \frac{2\omega^3}{\exp(\hbar\omega/kT) - 1} \cos\omega t$$
$$= \frac{6}{t^4} + \left\{ 2\left(\frac{\pi kT}{\hbar}\right)^4 \operatorname{csch}^2\left(\frac{\pi kTt}{\hbar}\right) \left[3 \operatorname{csch}^2\left(\frac{\pi kTt}{\hbar} + 2\right) \right] - \frac{6}{t^4} \right\},$$
$$\text{(F.121)}$$

where the first integral is of the type (F.67), and the second (F.43). As explained earlier, we obtain the same results for the other diagonal terms ($i = j$), both between two different electric fields and two different magnetic fields. Also, the expectation values between electric and magnetic fields vanish for all combinations of components. Therefore, we obtain the following results:

$$\begin{aligned}\left\langle 0 \,\middle|\, \overline{\mathbf{E}}_{Ti}(0, s - t/2)\, \overline{\mathbf{E}}_{Tj}(0, s + t/2) \middle|\, 0 \right\rangle^{\ddagger} &= \left\langle 0 \,\middle|\, \overline{\mathbf{B}}_{Ti}(0, s - t/2)\, \overline{\mathbf{B}}_{Tj}(0, s + t/2) \middle|\, 0 \right\rangle^{\ddagger} \\ &= \delta_{ij} \frac{4\hbar}{\pi c^3} \left(\frac{\pi k T}{\hbar}\right)^4 \left[\operatorname{csch}^4\left(\frac{\pi k T t}{\hbar}\right) + \frac{2}{3} \operatorname{csch}^2\left(\frac{\pi k T t}{\hbar}\right)\right],\end{aligned} \tag{F.122}$$

and

$$\begin{aligned}\left\langle 0 \,\middle|\, \overline{\mathbf{E}}_{Ti}(0, s - t/2)\, \overline{\mathbf{B}}_{Tj}(0, s + t/2) \middle|\, 0 \right\rangle^{\ddagger} &= \left\langle 0 \,\middle|\, \overline{\mathbf{B}}_{Ti}(0, s - t/2)\, \overline{\mathbf{E}}_{Tj}(0, s + t/2) \middle|\, 0 \right\rangle^{\ddagger} \\ &= 0, \quad i, j = 1, 2, 3.\end{aligned} \tag{F.123}$$

F.3.4 Comparison of Two Expectation Values

The vacuum expectation values have been evaluated for the case of electromagnetic vector fields in two different methods: one for an object under constant acceleration (hyperbolic motion) in ZPF, (F.115) and (F.116), and the other for an object in thermal radiation of temperature T, (F.122) and (F.123). We once again find similarities between these two expressions. However, the correspondence is not exact like the case of a scalar field, and it seems that the detector under hyperbolic motion in ZPF does not find the Planck spectrum.

This point has been further studied by Boyer[18], and it has been found that an oscillator under hyperbolic motion will experience a relativistic radiation reaction force related to its acceleration, and this extra term makes the oscillator respond with a

Planckian distribution with temperature

$$T = \frac{\hbar a}{2\pi k_B c}. \tag{F.124}$$

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